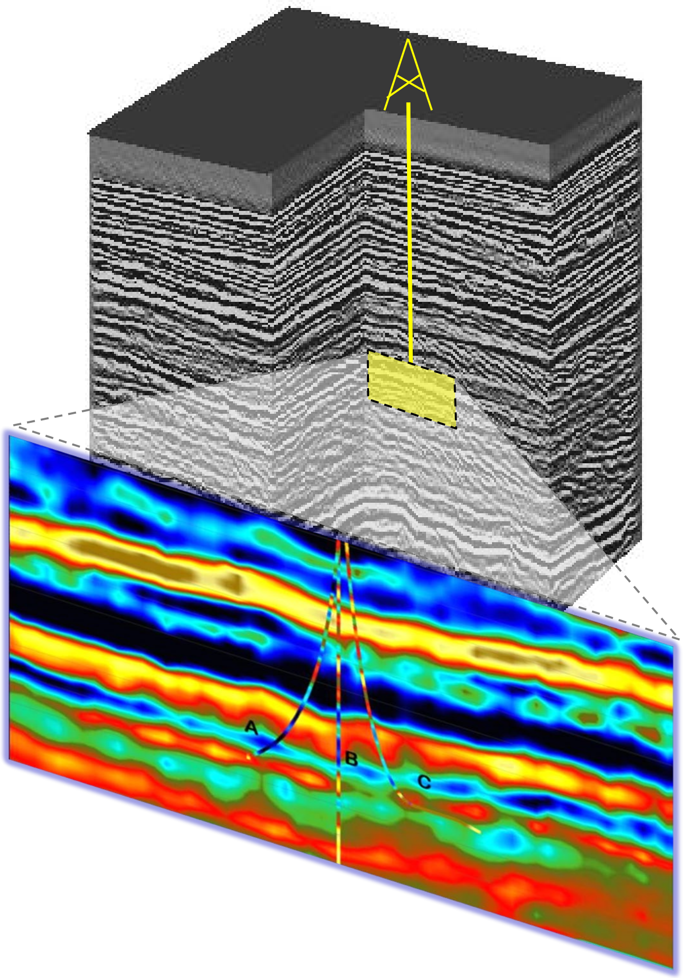
Introduction – Seismic Imaging



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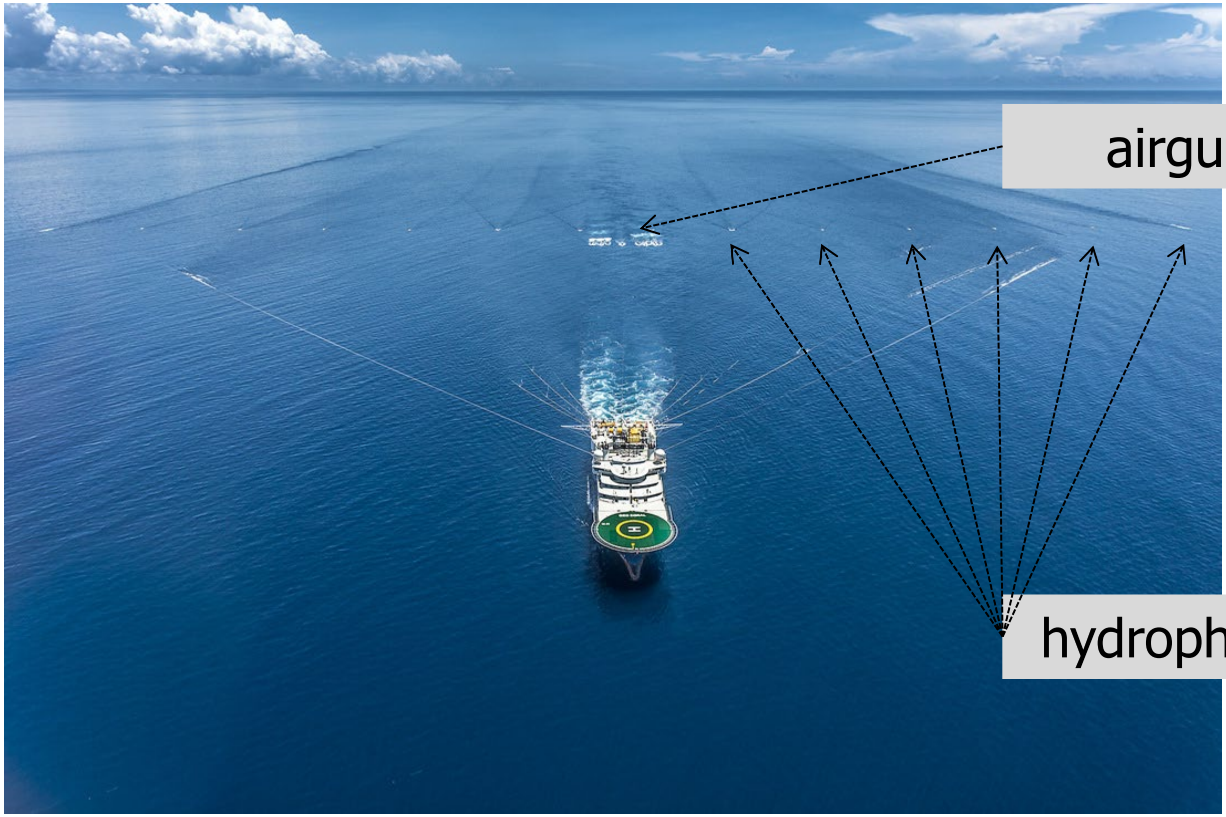
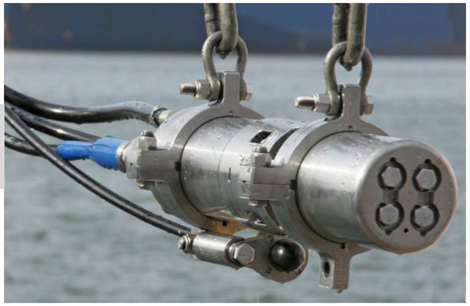
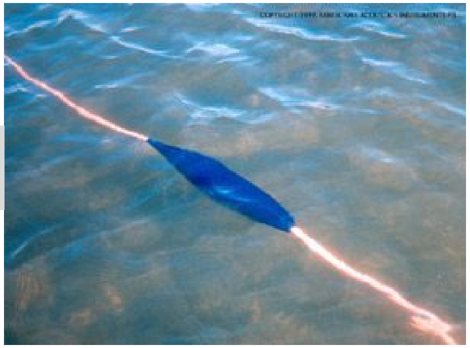
Introduction – Seismic Imaging







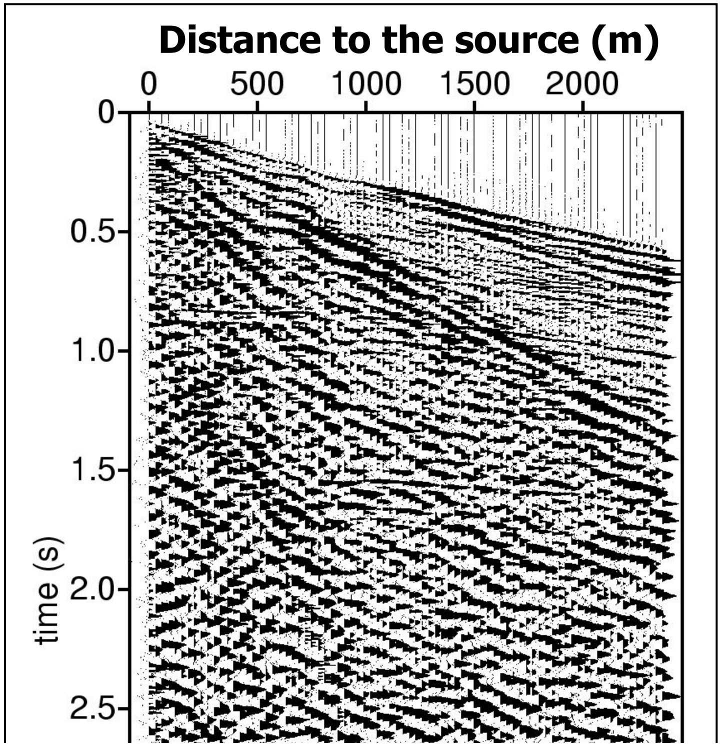
airgun

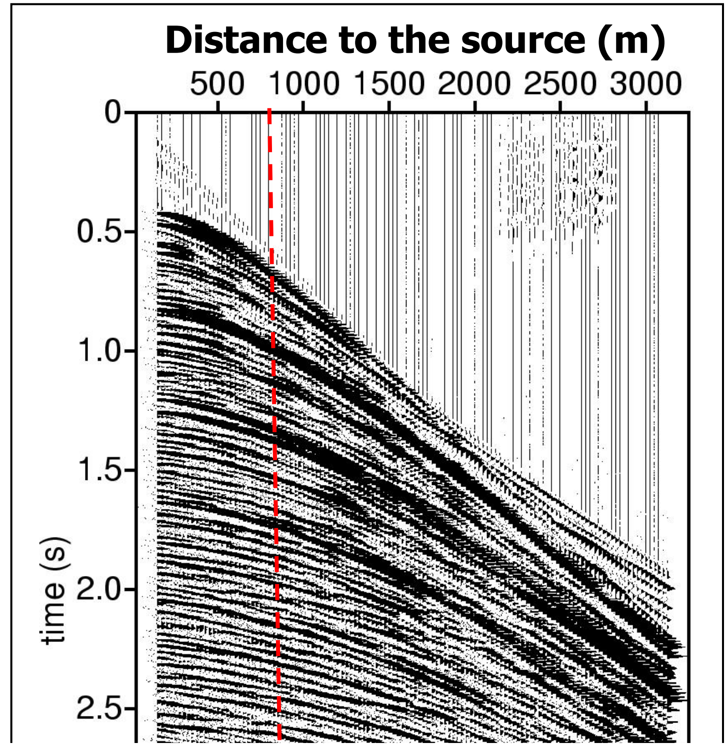
hydrophone

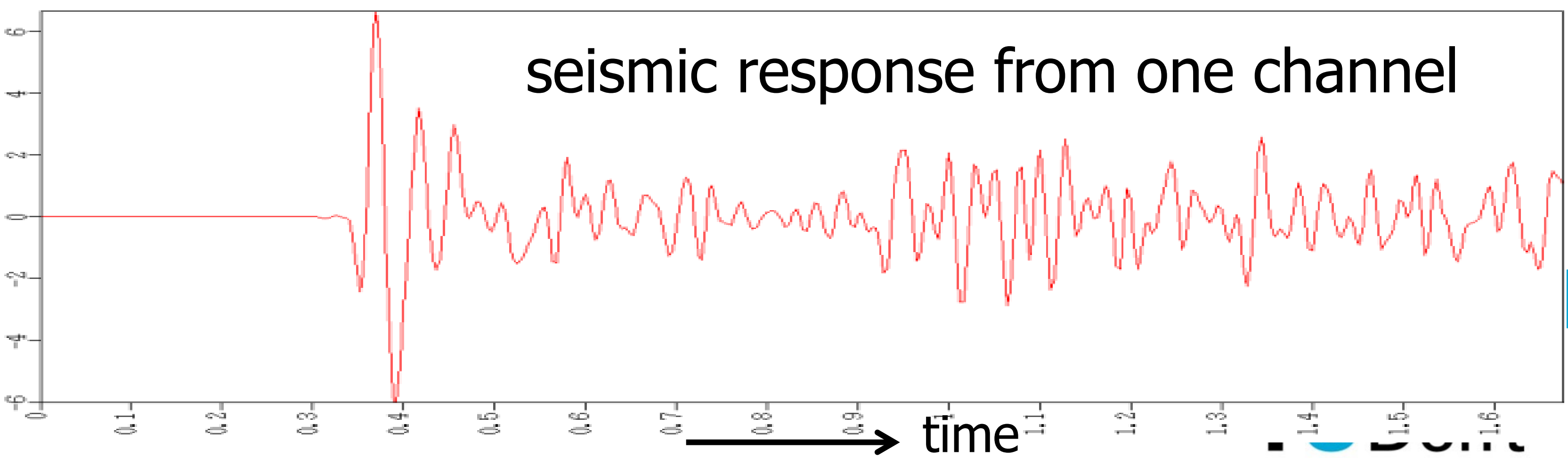
www.iagc.org
www.emma-technologies.com/
www.hydrophones.com/

Introduction – Seismic Imaging

Introduction – Seismic Imaging



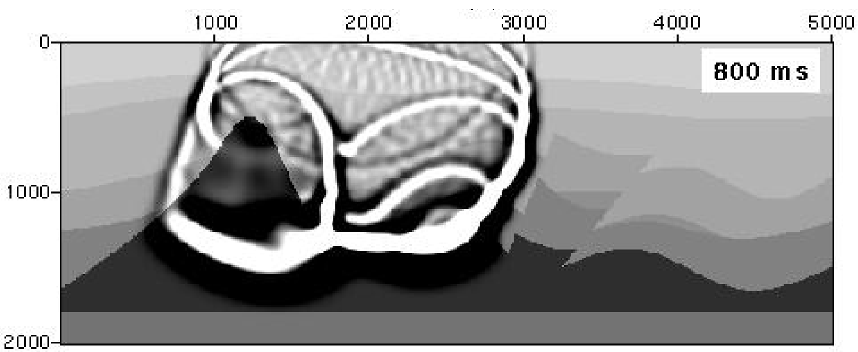




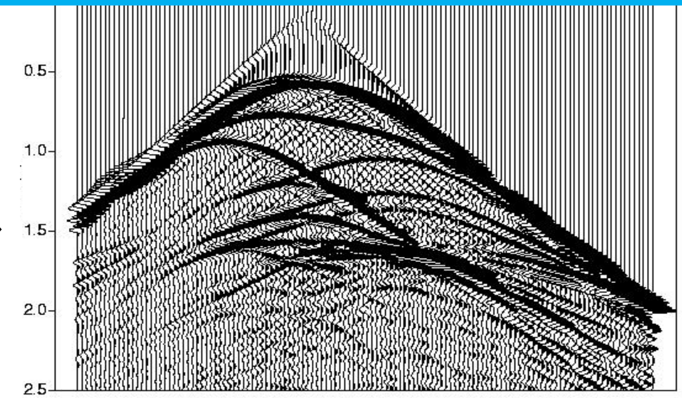
Introduction – Seismic Imaging

Obtain an image from a medium from acoustic reflection measurements

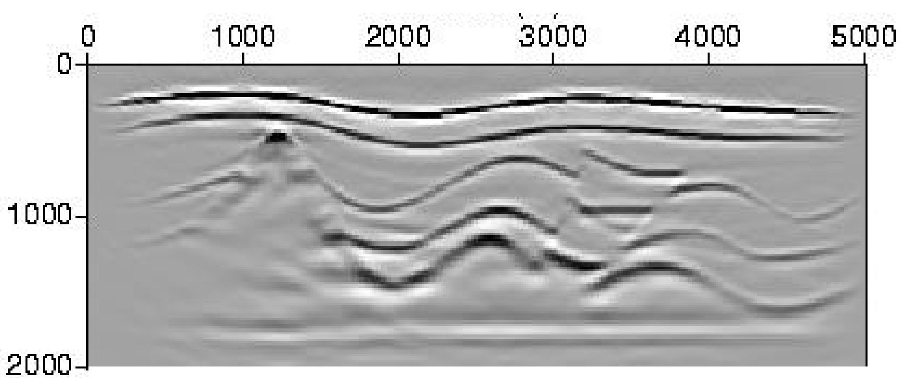
Transmit a sound wave into the medium



Measure the response with many channels

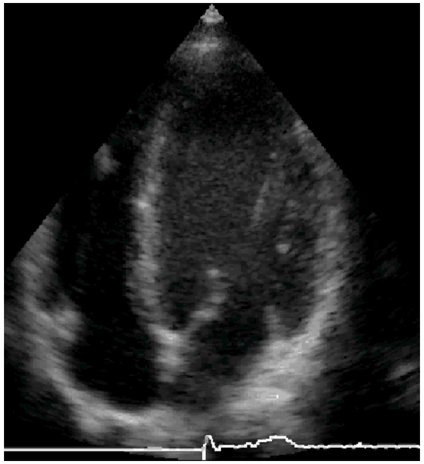


Reconstruct an image of the medium using wave theory



Vertical transect

Introduction – Medical Ultrasound



[1] …
[2] Erasmus MC, Rotterdam, the Netherlands
[3] http://langeproductions.com/

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Introduction – Nondestructive testing

[1] https://mtlabs.co.nz/services/non-destructive-testing/
[2] https://www.aerospacetestinginternational.com/features/introduction-to-non-destructive-testing.html

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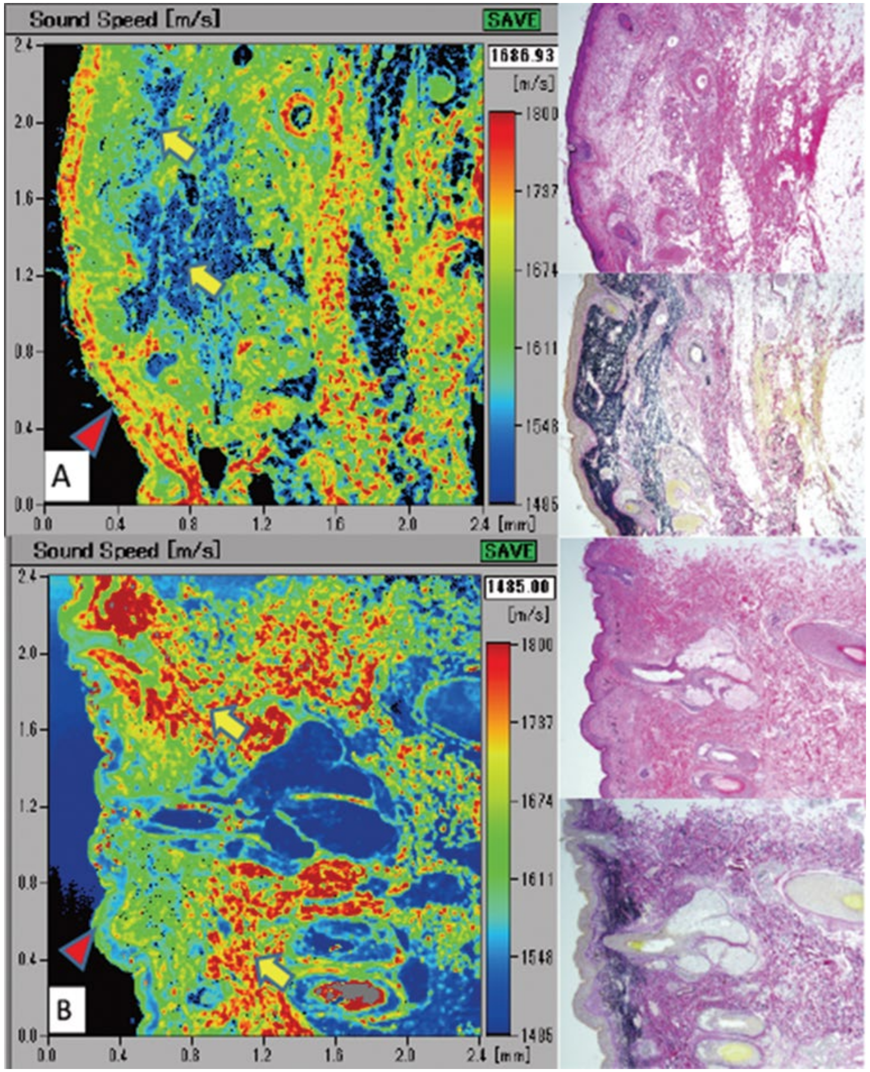
Introduction - Acoustic Microscopy

Comparative skin images of SOS



K. Miura, "Application of Scanning Acoustic Microscopy to Pathological Diagnosis," DOI: 10.5772/63405

Introduction – Applications Acoustical Imaging

- *>50 MHz*
- *10 MHz*
- *1 MHz*
- *100 kHz*
- *10 kHz*
- *1 kHz*
- *100 Hz*
- *10 Hz*
- *1 Hz*
- *0.1 Hz*

Acoustic Microscopy

Laminated materials (e.g. airplanes) → 0.5 cm deep

Inspections of welds → 0.5-5 cm deep

Medical ultrasound → 0.2-20 cm deep

Ship wreck detection → 1-20 m deep

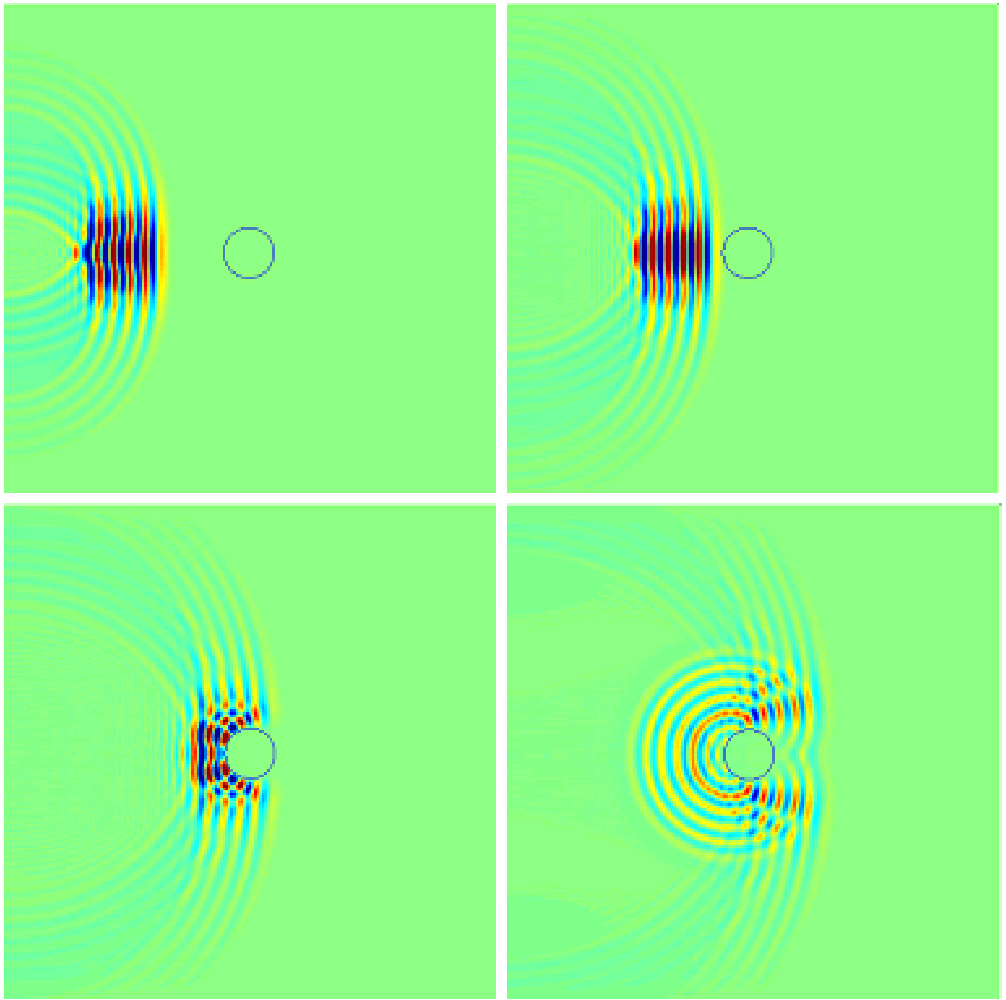
Near surface inspection → 5-500 m deep

Oil & gas exploration → 0.5-5 km deep

Tectonic imaging → 2-25 km depth

Global seismology → full earth

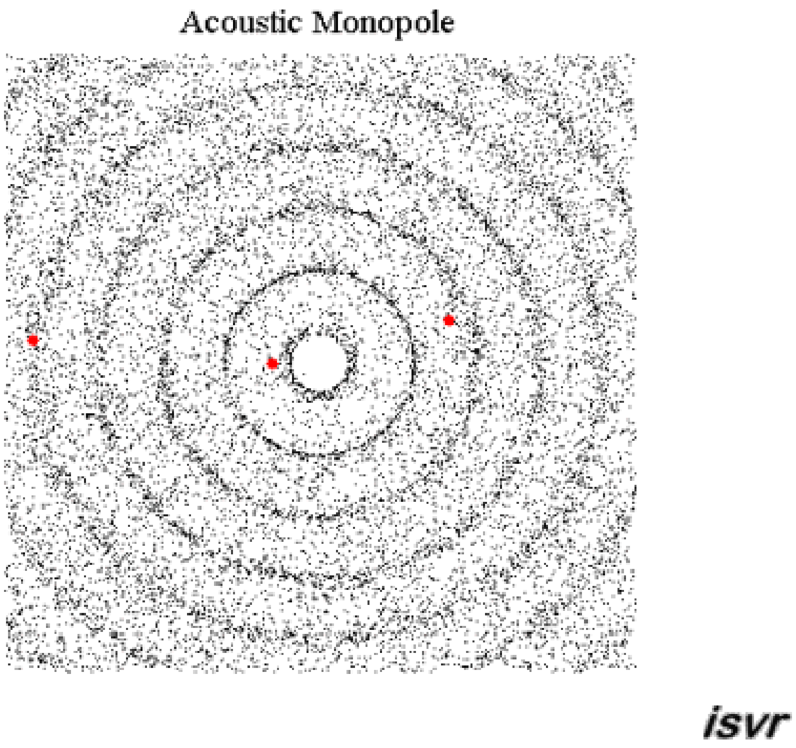
Introduction



[1] http://www.mathworks.com/matlabcentral/fileexchange/screenshots/1347/original.jpg

Introduction

Field radiated by a point source / acoustic monopole [1]



[1] http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-basics-pointsources.htm

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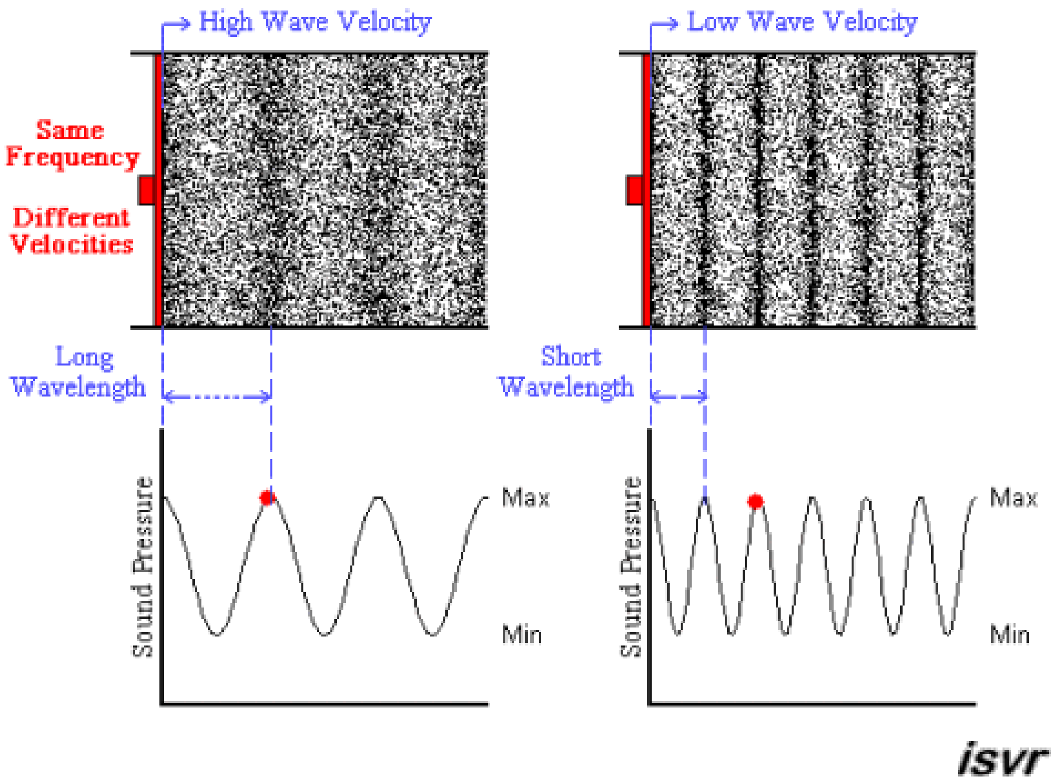
Introduction

Two plane waves with different velocities [1]



[1] http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-basics-frequency.htm

Acoustic Field Equations

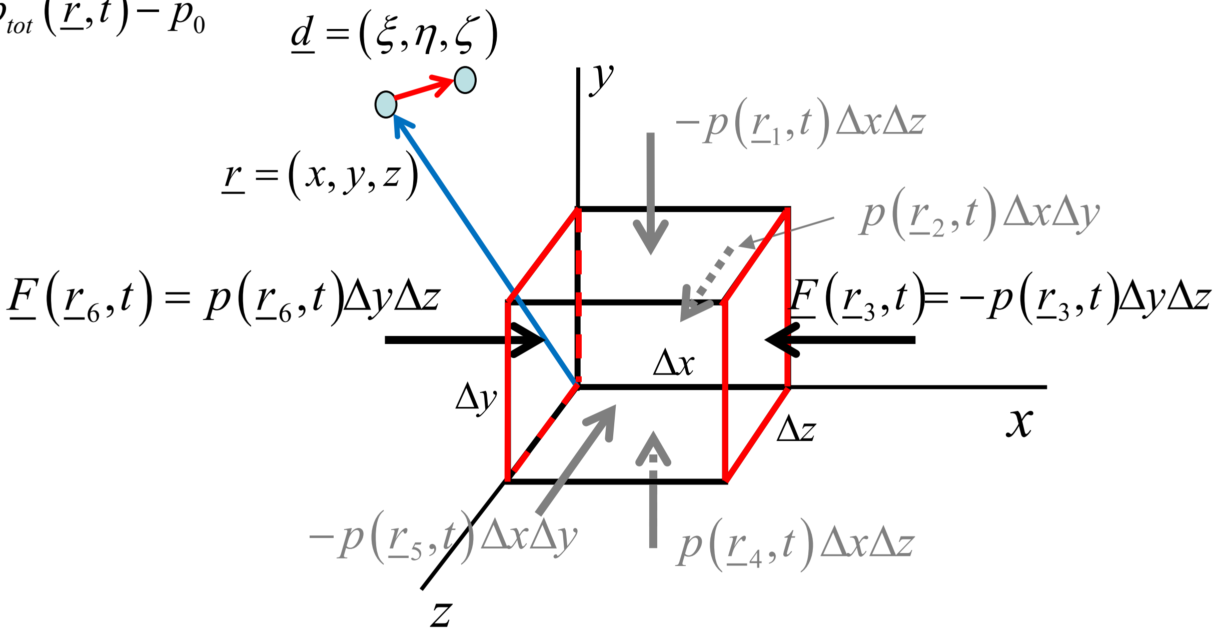
Volume: $\Delta V = \Delta x \Delta y \Delta z$

Location: $\underline{r} = (x, y, z)$

Displacement field: $\underline{d}(\underline{r},t) = [\xi(\underline{r},t), \eta(\underline{r},t), \zeta(\underline{r},t)]$

Velocity field: $\underline{v}(\underline{r},t) = \frac{d}{dt}\underline{d}(\underline{r},t)$

Pressure field: $p(\underline{r},t) = p_{tot}(\underline{r},t) - p_0$



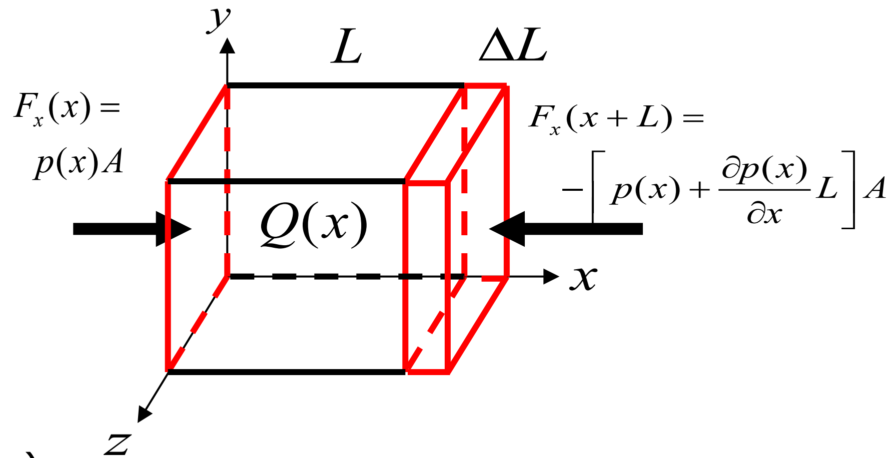
1-D Acoustic Field Equation – Hooke's Law

A fluid volume element ΔV is distorted by an excess pressure field $p(x,t)$ and volume injection $Q(x,t)$:

$$\Rightarrow \quad p_{tot}(x,t) = p_0 + p(x,t)$$

$$\Rightarrow \quad \Delta V \to \Delta V + A\Delta L$$

$$\Rightarrow \quad F = -k\frac{\Delta L}{L}$$ (Hooke's law: extension of a spring)



Amount of deformation depends on compressibility κ: $\frac{\Delta L}{L} = -\kappa p(x,t) + Q(x,t)$

Deformation can also be described by a displacement of the particle location $d_x(x,t)$: $\frac{\Delta L}{L} = \frac{\partial d_x(x,t)}{\partial x}$

$$\Rightarrow \quad -\kappa p(x,t) + Q(x,t) = \frac{\partial d_x(x,t)}{\partial x} \quad \xrightarrow{\frac{1}{\kappa}\frac{d}{dt} = \cancel{\frac{1}{\kappa}v\cdot\nabla} + \frac{1}{\kappa}\frac{\partial}{\partial t}} \quad \boxed{\frac{\partial p(x,t)}{\partial t} = -\frac{1}{\kappa}\frac{\partial v_x(x,t)}{\partial x} + \frac{1}{\kappa}q(x,t)}$$

Note that two linearizations are made: $\frac{d}{dt} = \cancel{v\cdot\nabla} + \frac{\partial}{\partial t}$ and $\kappa(p) = \kappa(p_0) + \cancel{(p-p_0)\left.\frac{\partial \kappa(p)}{\partial p}\right|_{p=p_0}}$

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3-D Acoustic Field Equation – Hooke's Law

Changing from one to three dimensions means that the volume will change in three dimensions and that the particles may move in the ξ, η and ζ – direction, hence

$$\frac{\delta \Delta V}{\Delta V} = \frac{\{\Delta x + \delta\xi\}\{\Delta y + \delta\eta\}\{\Delta z + \delta\zeta\} - \Delta x \Delta y \Delta z}{\Delta x \Delta y \Delta z} = \frac{\delta\xi \Delta y \Delta z + \delta\eta \Delta x \Delta z + \delta\zeta \Delta x \Delta y}{\Delta x \Delta y \Delta z} + O(\ldots) \approx \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z}$$

$$\Rightarrow \quad \boxed{\frac{\delta \Delta V}{\Delta V} = \nabla \cdot \underline{d}}$$

Consequently, Hooke's law will change into

$$\boxed{\frac{\partial p(\underline{r},t)}{\partial t} = -\frac{1}{\kappa} \nabla \cdot \underline{v}(\underline{r},t) + \frac{1}{\kappa} q(\underline{r},t)}$$

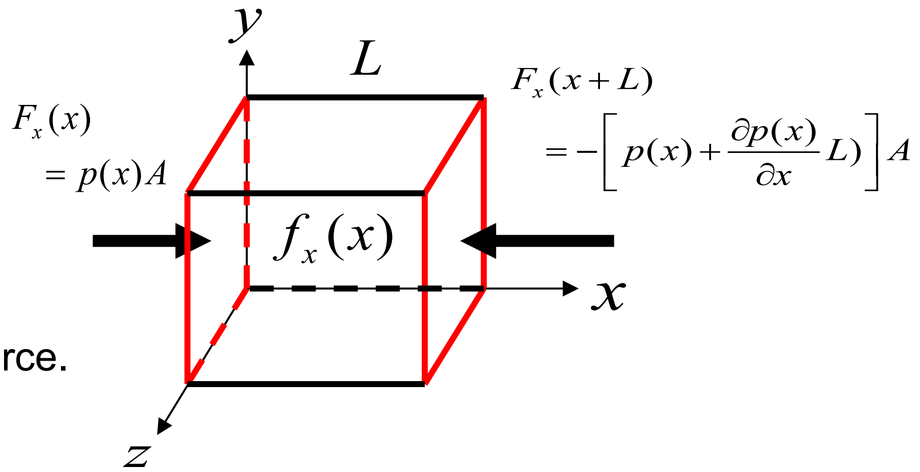
1-D Acoustic Field Equation – Newton's Law

With $p(x+L)=p(x)+\dfrac{\partial p}{\partial x}L$

we find for the net excess force ΔF on ΔV:

$$\Delta F=-A\left(\frac{\partial p(x,t)}{\partial x}L\right)+f_x(x,t)\Delta V$$

with $f_x(x,t)$ the volume source density of volume force.



Newton's Law, $F=ma$, for a fixed mass element $m=\Delta V\rho$ and $a=\dfrac{dv_x}{dt}=\cancel{v_x\dfrac{\partial v_x}{\partial x}}+\dfrac{\partial v_x}{\partial t}=\dfrac{\partial v_x}{\partial t}$ now reads:

$$\underbrace{-\Delta V\frac{\partial p(x,t)}{\partial x}+f_x(x,t)\Delta V}_{F}=\underbrace{\Delta V\rho}_{m}\underbrace{\frac{\partial v_x(x,t)}{\partial t}}_{a}$$

or:

$$\boxed{\frac{\partial p(x,t)}{\partial x}=-\rho\frac{\partial v_x(x,t)}{\partial t}+f_x(x,t)}$$

in 3-D the Newton's law reads:

$$\boxed{\nabla p(\underline{r},t)=-\rho\frac{\partial \underline{v}(\underline{r},t)}{\partial t}+\underline{f}(\underline{r},t)}$$

Note that two linearizations are made: $\dfrac{d}{dt}=\cancel{\underline{v}\cdot\nabla}+\dfrac{\partial}{\partial t}$ and $\rho(p)=\rho(p_0)+\cancel{(p-p_0)\dfrac{\partial\rho(p)}{\partial p}\Big|_{p=p_0}}$

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Acoustic Field Equations

The obtained acoustic field equations read

Hooke's law: $$\frac{\partial p(\underline{r},t)}{\partial t} = -\frac{1}{\kappa}\nabla\cdot\underline{v}(\underline{r},t) + \frac{1}{\kappa}q(\underline{r},t)$$ (equation of deformation)

Newton's law: $$\nabla p(\underline{r},t) = -\rho\frac{\partial \underline{v}(\underline{r},t)}{\partial t} + \underline{f}(\underline{r},t)$$ (equation of motion)

This set of equations show large similarities with Maxwel Equations

$$\nabla\cdot\underline{E} = \frac{1}{\varepsilon_0}\rho_f \qquad \nabla\cdot\underline{B} = 0$$

$$\nabla\times\underline{E} = -\frac{\partial\,\underline{B}}{\partial t} \qquad \nabla\times\underline{B} = \mu\sigma\underline{E} + \mu\varepsilon\frac{\partial\,\underline{E}}{\partial t}$$

Wave Equation

The acoustic field equations may be combined to obtain (in the absense of sources):

a) a scalar wave equation for the pressure field:

$$\left.\begin{aligned}\rho\kappa\frac{\partial}{\partial t}\left[\frac{\partial p(\underline{r},t)}{\partial t}\right] &= \rho\kappa\frac{\partial}{\partial t}\left[-\frac{1}{\kappa}\nabla\cdot\underline{v}(\underline{r},t)\right]\\ \nabla\cdot\left[-\nabla p(\underline{r},t)\right] &= \nabla\cdot\left[\rho\frac{\partial\underline{v}(\underline{r},t)}{\partial t}\right]\end{aligned}\right\}\Rightarrow \boxed{\nabla^2 p(\underline{r},t)-\rho\kappa\frac{\partial^2 p(\underline{r},t)}{\partial t^2}=0}$$

$$\rho\kappa = \frac{1}{c^2}$$

remember: $\nabla^2\underline{E}(\underline{r},t)-\frac{1}{c^2}\frac{\partial^2\underline{E}(\underline{r},t)}{\partial t^2}=0$

$\nabla^2\underline{B}(\underline{r},t)-\frac{1}{c^2}\frac{\partial^2\underline{B}(\underline{r},t)}{\partial t^2}=0$

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Wave Equation

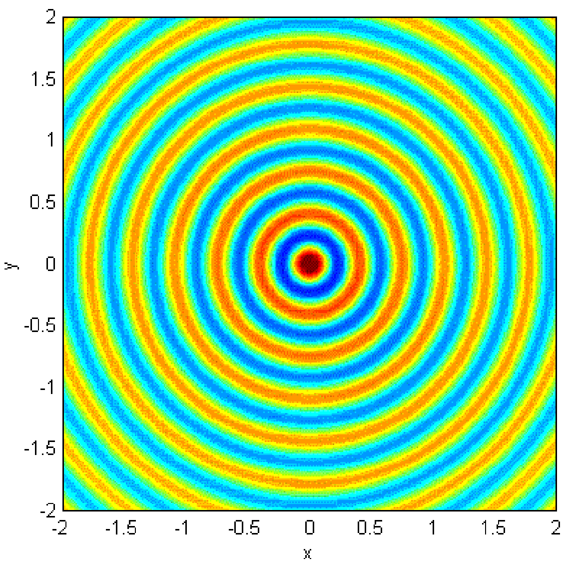
There are two types of sources, which can generate an acoustic field

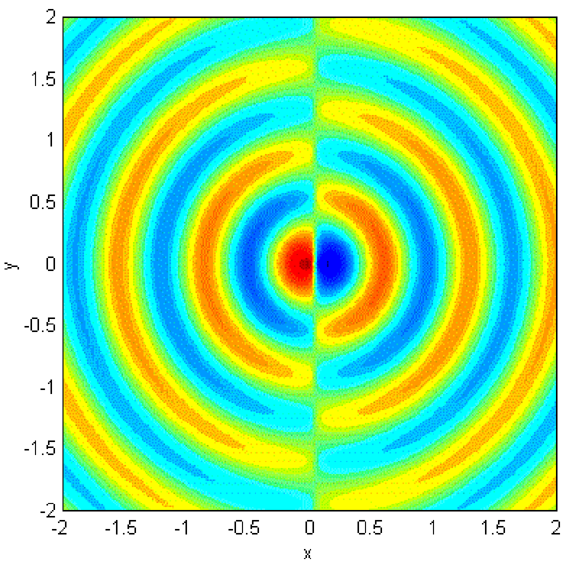
1) A volume source density of injection rate: $q(\underline{r},t)$ [s^{-1}]

2) A volume source density of volume source: $\underline{f}(\underline{r},t)$ [N/m^3]

Hooke's law: $$\frac{\partial p(\underline{r},t)}{\partial t} = -\frac{1}{\kappa}\nabla\cdot\underline{v}(\underline{r},t) + \frac{1}{\kappa}q(\underline{r},t),$$ (equation of deformation)

Newton's law: $$\nabla p\left(\underline{r},t\right) = -\rho\frac{\partial\underline{v}\left(\underline{r},t\right)}{\partial t} + \underline{f}(\underline{r},t).$$ (equation of motion)





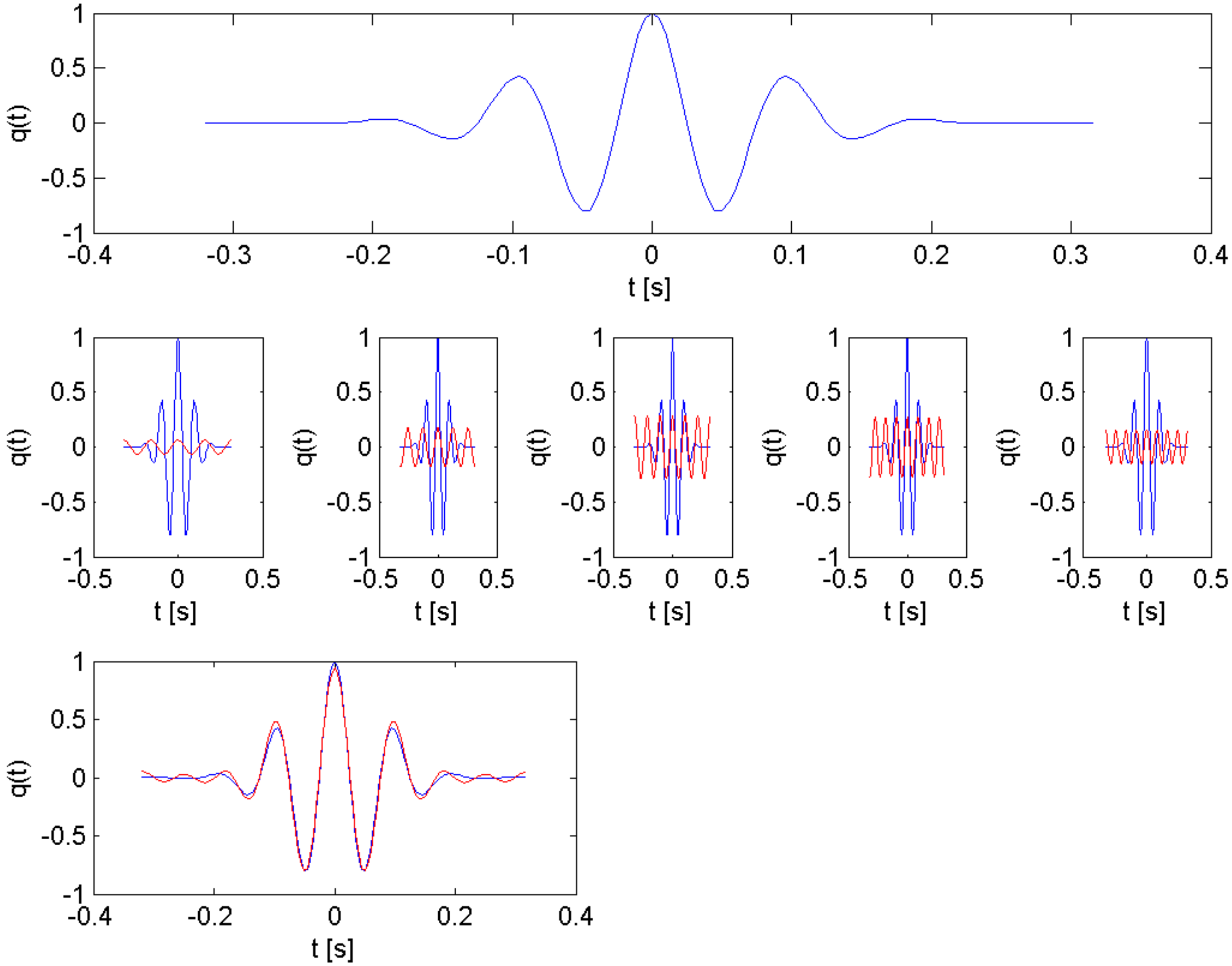
[1] http://en.wikibooks.org/wiki/Aeroacoustics/Acoustic_Sources

Helmholtz Equation

Any pulse can be described by a combination of sine and cosine functions:

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{F}(\omega)\, e^{+i\omega t}\, d\omega$$



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Helmholtz Equation

Definition of the temporal Fourier transform of a function $g(\underline{r},t): \hat{g}(\underline{r},\omega) = \int_{-\infty}^{\infty} g(\underline{r},t)\, e^{-i\omega t} dt$

Fourier transformation of the wave equation $\nabla^2 p(\underline{r},t) - \frac{1}{c^2}\frac{\partial^2 p(\underline{r},t)}{\partial t^2} = 0$

using: $\int_{-\infty}^{\infty} \left(\frac{\partial g(\underline{r},t)}{\partial t} \right) e^{-i\omega t} dt = i\omega\, \hat{g}(\underline{r},\omega)$ yields the Helmholtz equation

$$\boxed{\nabla^2 \hat{p}(\underline{r},\omega) + \frac{\omega^2}{c^2} \hat{p}(\underline{r},\omega) = 0}$$

Frequency domain Acoustic Field Equations

In the frequency domain, the obtained acoustic field equations equal

Hooke's law: $$i\omega\hat{p}(\underline{r},\omega) = -\frac{1}{\kappa}\nabla\cdot\hat{\underline{v}}\left(\underline{r},\omega\right),$$

Newton's law: $$-\nabla\hat{p}\left(\underline{r},\omega\right) = i\omega\rho\hat{\underline{v}}\left(\underline{r},\omega\right).$$

Spherical Waves

Transforming the Helmholtz equation to polar coordinates
for spherical symmetric solutions yields

$$\nabla^2 \hat{p}(\underline{r},\omega) + \frac{\omega^2}{c^2}\hat{p}(\underline{r},\omega) = -\hat{S}(\underline{r},\omega) \quad \Rightarrow \quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\hat{p}(r,\omega)\right) + \frac{\omega^2}{c^2}\hat{p}(r,\omega) = -\delta(\underline{r})\hat{S}(\omega).$$

The most general solution of this equation equals:

$$\hat{p}(r,\omega) = \hat{A}(\omega)\frac{e^{-i\omega r/c_0}}{r} + \hat{B}(\omega)\frac{e^{i\omega r/c_0}}{r}, \quad \text{for } r \neq 0.$$

Why *exp[…]* ?

Why *1/r* ?

In practise, with acoustics only the outgoing spherical wave is encountered. In literature, the spherical wave created by a Dirac-delta source is referred to as

Green's function $\hat{G}(\underline{r},\omega)$, or the impulse response of the medium:

$$\boxed{\hat{G}(\underline{r},\omega) = \frac{e^{-i\omega|\underline{r}|/c_0}}{4\pi|\underline{r}|}}$$

Green’s functions

1-D: $$\left(\frac{d^2}{dx^2}+k^2\right)\hat{G}(x)=-\delta\left(x-a\right) \rightarrow \hat{G}(x)=\frac{i}{2k}e^{-ikr} \quad \text{with } r=|x-a|$$

2-D: $$\left(\nabla^2+k^2\right)\hat{G}(\vec{x})=-\delta\left(\vec{x}-\vec{a}\right) \rightarrow \hat{G}(\vec{x})=-\frac{i}{4}H_0^{(2)}(kr) \quad \text{with } r=|\vec{x}-\vec{a}|$$

3-D: $$\left(\nabla^2+k^2\right)\hat{G}(\vec{x})=-\delta\left(\vec{x}-\vec{a}\right) \rightarrow \hat{G}(\vec{x})=\frac{1}{4\pi}\frac{e^{-ikr}}{r} \quad \text{with } r=|\vec{x}-\vec{a}|$$

Weak form the Greens function

The 2-D and 3-D Greens function's are singular for $r=0$. To gives rise to numerical problems when implementing these Green's functions. These problems are solved by using there spherical mean, which is not singular at $r=0$.

2-D: $$\hat{G}(\vec{x}) = -\frac{i}{4} H_0^{(2)}(kr) \quad \rightarrow \quad \begin{cases} \dfrac{i}{2ka} J_1(ka) H_0^{(1)}(kr) & r \geq a \\[2ex] \dfrac{i}{2ka}\left[H_1^{(1)}(ka) + \dfrac{2i}{\pi ka} \right] & r = 0 \end{cases}$$

3-D: $$\hat{G}(\vec{x}) = \frac{1}{4\pi} \frac{e^{-ikr}}{r} \quad \rightarrow \quad \begin{cases} \dfrac{3e^{-ikr}}{4k^3 \pi a^3 r}\left[\sin(ka) - ka\cos(ka) \right] & r \geq a \\[2ex] \dfrac{3}{4k^2 \pi a^3}\left[(1+ika)e^{-ika} - 1 \right] & r = 0 \end{cases}$$

Attenuation

Attenuation is mainly caused by absorption (the transformation of acoustic energy into e.g. heat), geometrical spreading and scattering.

There are various ways to include absorption (the transformation of acoustic energy into e.g. heat) in the field equations. One approach is to use memory or relaxation functions.

In the presence of these memory functions, the acoustic field equations read

Hooke's law: $$\frac{\partial \kappa(t) *_t p(\underline{r},t)}{\partial t} = -\nabla \cdot \underline{v}(\underline{r},t) + q(\underline{r},t)$$ (equation of deformation)

Newton's law: $$\nabla p(\underline{r},t) = -\frac{\partial \rho(t) *_t \underline{v}(\underline{r},t)}{\partial t} + \underline{f}(\underline{r},t)$$ (equation of motion)

with $\kappa(t) = \kappa_0 \delta(t) + \kappa_m(t)$

$\rho(t) = \rho_0 \delta(t) + \rho_m(t).$

The mechanism underlying attenuation is amongst others related to the viscocity of the medium.

Note that in general, attenuation is frequency depenent effect and will give rise to dispersion.

Attenuation and Kramers-Kronig relations

Defining the relaxation functions for the field equations is a difficult process.

Measurements show that for biomedical tissues attenuation may be described via a power law. However, care has to be taken as these relaxation functions should meet the requirements set by nature;

- they should be real valued in the time domain and,
- they should meet the requirements set by causality.

From these requirements, the Kramers-Kronig or generalized dispersion relations are derived :

$$\kappa(\omega) = \kappa_R(\omega) + i\kappa_I(\omega)$$

Complex wave numbers !

with

$$\kappa_R(\omega) = \frac{2}{\pi} p(\omega) \int_0^\infty \frac{\omega' \kappa_I(\omega')}{(\omega')^2 - \omega^2} d\omega'$$

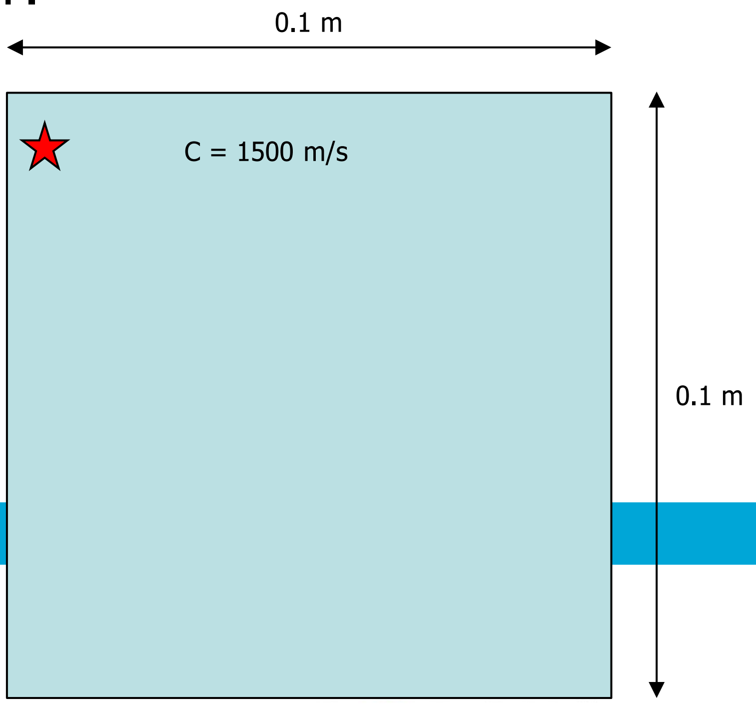
and

$$\kappa_I(\omega) = -\frac{2}{\pi} p(\omega) \int_0^\infty \frac{\omega' \kappa_R(\omega')}{(\omega')^2 - \omega^2} d\omega'$$

See e.g. M. O'Donnell, E. T. Jaynes, and J. G. Miller, "Kramers-Kronig relationship between ultrasonic attenuation and phase velocity," *J. Acoust. Soc. Amer.* 69(3), 696-701 (1981).

Exercise

- Compute the pressure field generated by a point source (volume source density of injection rate)
  - Centre frequency Gaussian pulse f_0 = 1 MHz;
  - Medium water (c = 1500 m/s, ρ = 1000 kg/m^3);
  - Locate the point source in the centre of a volume (e.g. 0.1 m x 0.1 m x 0.0015 m).
- Compute the pressure field in the frequency domain and transform the resulting field to the time domain using FFT.
- Δx = ?
- Δt = ?
- Nt = ?



Exercise

Derive the Green's function $\hat{G}(\underline{r},\omega)$, or the impulse response of the medium:

$$\hat{G}(\underline{r},\omega) = \frac{e^{-i\omega|\underline{r}|/c_0}}{4\pi|\underline{r}|}$$

Hint:

1) Use as a starting point the Helmholtz equation, i.e. $\nabla^2\hat{G}(\underline{r},\omega) + \frac{\omega^2}{c^2}\hat{G}(\underline{r},\omega) = -\delta(\underline{r})$.

2) Transform this equation to the spatial Fourier domain to find an expression for $\tilde{\hat{G}}(\underline{k},\omega)$.

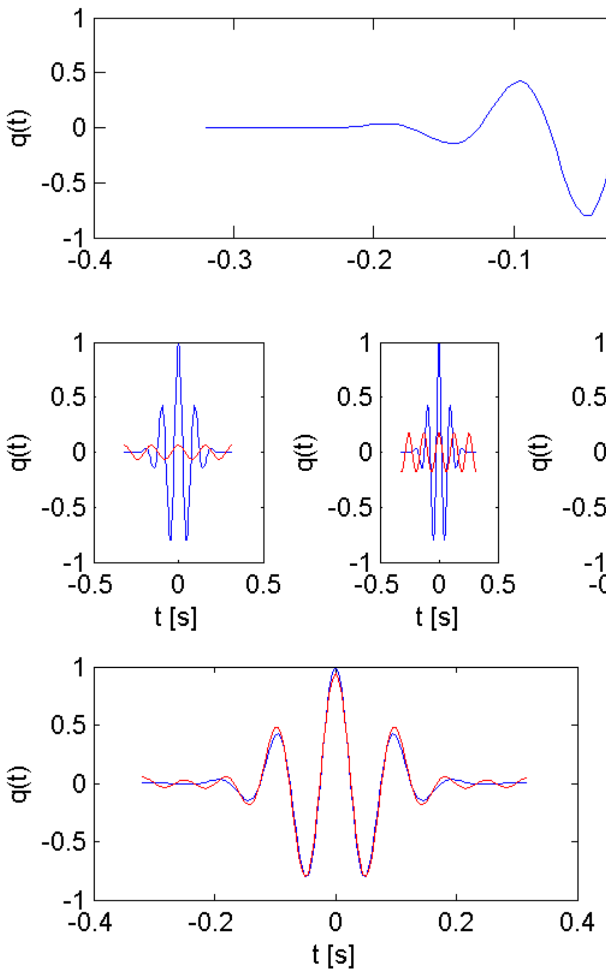
3) Transform the resulting expression for $\tilde{\hat{G}}(\underline{k},\omega)$ back to the spatial domain using spherical coordinates to find an expression for $\hat{G}(\underline{r},\omega)$.
For this part you need to use a contour integration around a complex pole.

Plane Waves

Any pulse can be described by a combination of sine and cosine functions:

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{F}(\omega)\, e^{+i\omega t}\, d\omega$$



Plane Waves – Acoustic Impedance

The concept of describing a pulse using Fourier series can be extended to 1-D, 2-D and 3-D wave fields, leading to the introduction of plane waves.

General solutions of the Helmholtz equation, $\nabla^2 \hat{p}(\underline{r},\omega) + \frac{\omega^2}{c^2}\hat{p}(\underline{r},\omega) = 0$,

in terms of plane waves equal $\boxed{\hat{p}(\underline{r},\omega) = \sum_{\underline{k}} F_{\underline{k}}(\omega) e^{-i\underline{k}\cdot\underline{r}}}$, with wave vector $\underline{k}$ of length $|\underline{k}| = \frac{\omega}{c}$.

Calculating the velocity field that goes along with these pressure plane waves yields

$$\left.\begin{aligned} -\nabla \hat{p}(\underline{r},\omega) &= i\omega\rho_0 \hat{\underline{v}}(\underline{r},\omega) \\ \hat{p}(\underline{r},\omega) &= \sum_{\underline{k}} F_{\underline{k}}(\omega) e^{-i\underline{k}\cdot\underline{r}} \end{aligned}\right\} \Rightarrow \quad \sum_{\underline{k}} i\underline{k}\, F_{\underline{k}}(\omega) e^{-i\underline{k}\cdot\underline{r}} = i\omega\rho \hat{\underline{v}}(\underline{r},\omega) \quad \Rightarrow \quad \hat{\underline{v}}(\underline{r},\omega) = \frac{1}{\rho c} \sum_{\underline{k}} \frac{\underline{k}}{|\underline{k}|} F_{\underline{k}}(\omega) e^{-i\underline{k}\cdot\underline{r}}$$

1-D : $\Rightarrow$ $\boxed{\hat{p}(x,\omega) = \rho c\, \hat{v}(x,\omega) = Z\, \hat{v}(x,\omega)}$, where Z is refered to as the acoustic impedance.

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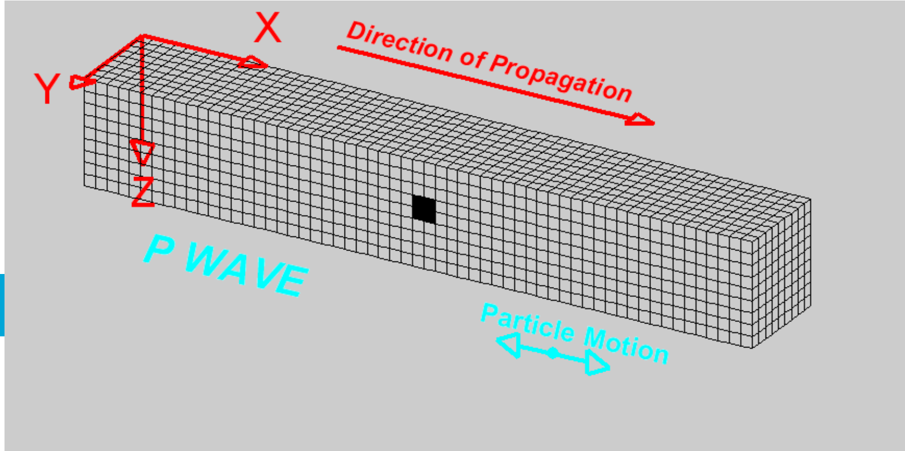
Longitudinal wave propagation

If we take a pressure wave which travels only in the x-direction, we obtain the following expression:

$$\hat{p}\left(\underline{r},\omega\right) = \sum_{\underline{k}} F_{\underline{k}}(\omega) e^{-i\underline{k}\cdot\underline{r}} = F(\omega) e^{-i\frac{\omega}{c}x}$$

For this expression, the velocity wave equals $\hat{\underline{v}}\left(\underline{r},\omega\right) = \frac{1}{\rho c}\sum_{\underline{k}} \frac{\underline{k}}{\left|\underline{k}\right|} F_{\underline{k}}(\omega) e^{-i\underline{k}\cdot\underline{r}} = \frac{1}{\rho c} F(\omega) e^{-i\frac{\omega}{c}x} = \hat{p}\left(\underline{r},\omega\right).$

Generalising this to three dimensions shows that the particle movement is in the direction of propagation, leading to longitudinal waves.



Impedance Single Outgoing Spherical Wave

Outgoing spherical wave:

$$p(r,t) = \frac{f(t - r/c)}{r}$$

In the frequency domain:

$$\hat{p}(r,\omega) = F(\omega)\frac{e^{-i\omega r/c}}{r}$$

Calculate particle velocity from Newton's law:

$$\underline{\hat{v}}(r,\omega) = -\frac{1}{i\omega\rho}\nabla\hat{p}(r,\omega) \quad , \quad \nabla\hat{p}(r,\omega) = \frac{\partial\hat{p}}{\partial r}\underline{\hat{r}}$$

$$\underline{\hat{v}}(r,\omega) = \frac{1}{\rho c}\left(1 + \frac{1}{i\omega r/c}\right)F(\omega)\frac{e^{-i\omega r/c}}{r}\underline{\hat{r}} = \frac{1}{\rho c}\left(1 + \frac{1}{i\omega r/c}\right)\hat{p}(r,\omega)\underline{\hat{r}}$$

Impedance: $Z(r,\omega) = \dfrac{\hat{p}(r,\omega)}{\hat{v}(r,\omega)} = \dfrac{\rho c}{1 + \dfrac{1}{i\omega r/c}}$, $\left(\underline{\hat{v}}(r,\omega) = \hat{v}(r,\omega)\underline{\hat{r}} \right)$

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Near and Far Field approximations for Spherical Waves

The impedance Z equals $Z(r,\omega)=\dfrac{\hat{p}(r,\omega)}{\hat{v}(r,\omega)}=\dfrac{\rho c}{1+\dfrac{1}{i\omega r/c}}=\dfrac{\rho c\left(1+\dfrac{ic}{\omega r}\right)}{1+\dfrac{c^2}{\omega^2 r^2}}$

In the far field ($\omega r/c >> 1$) we have:

$$\hat{p}(r,\omega)\approx\rho c\,\hat{v}(r,\omega)$$

The impedance becomes real valued and hence *resistive.* In the far field spherical waves behave like plane waves.

In the near field ($\omega r/c << 1$) we have:

$$\hat{p}(r,\omega)\approx i\omega\rho r\,\hat{v}(r,\omega)$$

This gives a predominantly imaginary impedance or a *reactive* impedance.

Now the pressure and the velocity wave field are out of phase!

Energy of Acoustic Wave

- Apparently, for plane acoustic waves, pressure and particle velocity are in phase. The proportionality factor $\rho_0 c$ is called the acoustic impedance of the medium:

$$Z = \rho_0 c$$

- Kinetic energy density is:

$$\delta V'_{kin} = \tfrac{1}{2}\rho_0 v^2$$

Using: $p = \rho_0 c\, v$ we have: $\delta V'_{kin} = \tfrac{1}{2}\dfrac{p^2}{\rho_0 c^2} = \tfrac{1}{2}\kappa p^2 = \delta V'_{pot}$

- Because particle velocity and pressure are in phase, work is done. The work done per unit area, per time-unit, is the power carried by a propagating acoustic wave through a unit surface area perpendicular to the direction of propagation.

Energy of Acoustic Wave & Parseval's Theorem

The instantaneous intensity I of an acoustic wave is equal to the energy flux through an unit area with normal $\underline{n}$, hence

$$I(\underline{r},t) = p(\underline{r},t)\underline{v}(\underline{r},t)\cdot\underline{n}$$

What about dimensions?

The time-averaged intensity is equal to

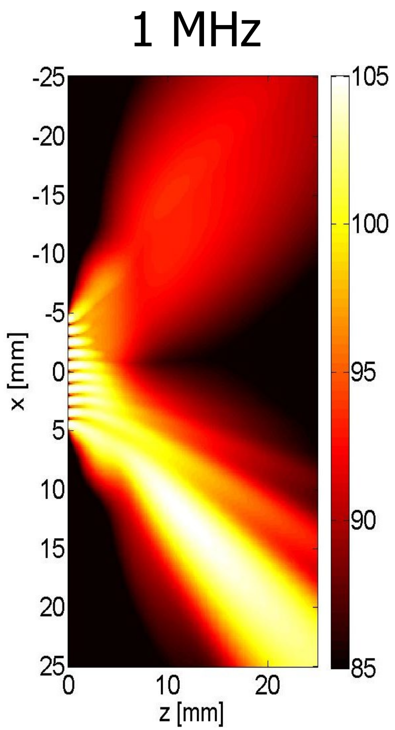
$$I_{av} = \frac{1}{T}\int_0^T I(\underline{r},t)\,dt = \frac{1}{T}\int_0^T p(\underline{r},t)\underline{v}(\underline{r},t)\cdot\underline{n}\,dt.$$

Because both $p(\underline{r},t)$ and $\underline{v}(\underline{r},t)$ are real valued functions, we use Parseval's theorem to express the total energy flow through an unit area in the frequecy domain as

$$J(\underline{r}) = \int_{-\infty}^{\infty} p(\underline{r},t)\underline{v}(\underline{r},t)\cdot\underline{n}\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{p}(\underline{r},\omega)\left[\hat{\underline{v}}(\underline{r},\omega)\right]^{*}\cdot\underline{n}\,d\omega$$

Nonlinear Acoustics

Arrays are used to steer the beam into a certain direction.
Due to the finite size of the elements and the spacing in between elements, side and grating lobes will occur, leading to a blurring of the image.



Nonlinear Acoustics

To derive the linear wave equation, three approximations were made

- the volume density of mass was taken to be constant (pressure independent):

$$\rho_0(p) = \rho_0(p_0) + (p - p_0)\left.\left(\frac{\partial \rho_0(p)}{\partial p}\right)\right|_{p=p_0},$$

- the bulk modulus (or compressibility) was taken to be constant (pressure independent):

$$\kappa_0(p) = \kappa_0(p_0) + (p - p_0)\left.\left(\frac{\partial \kappa_0(p)}{\partial p}\right)\right|_{p=p_0},$$

- the convection term of the material derivative was taken to be zero:

$$\frac{dp(\underline{r},t)}{dt} \equiv \frac{\partial p(\underline{r},t)}{\partial t} + \underline{v} \cdot \nabla p(\underline{r},t)$$

Nonlinear Acoustics

For linear acoustics, experiments show that the previous assumptions are valid.

However, for high amplitude pressure fields, this approximation is no longer valid, moreover the volume density of mass $\rho(p)$ may be approximated by

$$\left.\begin{aligned}
&\rho(p) = \rho_0 + (p - p_0)\left.\frac{\partial \rho}{\partial p}\right|_{p-p_0} \\
&p_0\left(\Delta V\right)^{\gamma} = const \;\Rightarrow\; p_0\left(\rho_0\right)^{-\gamma} = p\left(\rho\right)^{-\gamma} \;\Rightarrow\; \frac{\partial \rho}{\partial p} = \frac{\rho}{\gamma p} \\
&\gamma p_0 = \frac{1}{\kappa_0}
\end{aligned}\right\} \Rightarrow \boxed{\rho(p) = \rho_0\left[1 + \kappa_0 (p - p_0)\right]}$$

A similar expression may be obtained for the compressibilitiy $\kappa(p)$

$$\boxed{\kappa(p) = \kappa_0\left[1 + \kappa_0 (1 - 2\beta)(p - p_0)\right]}$$

with β the coëfficient of non-linearity.

Speed of Sound

Combination of the second order approximations

$$\frac{1}{c(p)^2} = \kappa(p)\rho(p)$$

$$= \kappa_0\rho_0\left[1+\kappa_0 p)\right]\left[1+\kappa_0(1-2\beta)p\right]$$

$$= \kappa_0\rho_0\left[1+2\kappa_0(1-\beta)p+\kappa_0^2(1-2\beta)p^2\right].$$

Typical values for β vary from 3.6 (water) to 10 (methane).

Hence, for

- increasing pressure we observe an increase in speed of sound c,
- decreasing pressure we observe an decrease in speed of sound c,

This will lead to a change of the shape of the waveform of the wavefield.

Nonlinear Propagation

Propagation history of an intense acoustic waveform that is sinusoidal at the source.

(a) $x = 0$: source waveform,

(b) distortion becoming noticeable,

(c) $x = x_s$: shock formation,

(d) $x = (\pi/2)\, x_s$ maximum shock amplitude,

(e) $x = 3\, x_s$ full sawtooth shape,

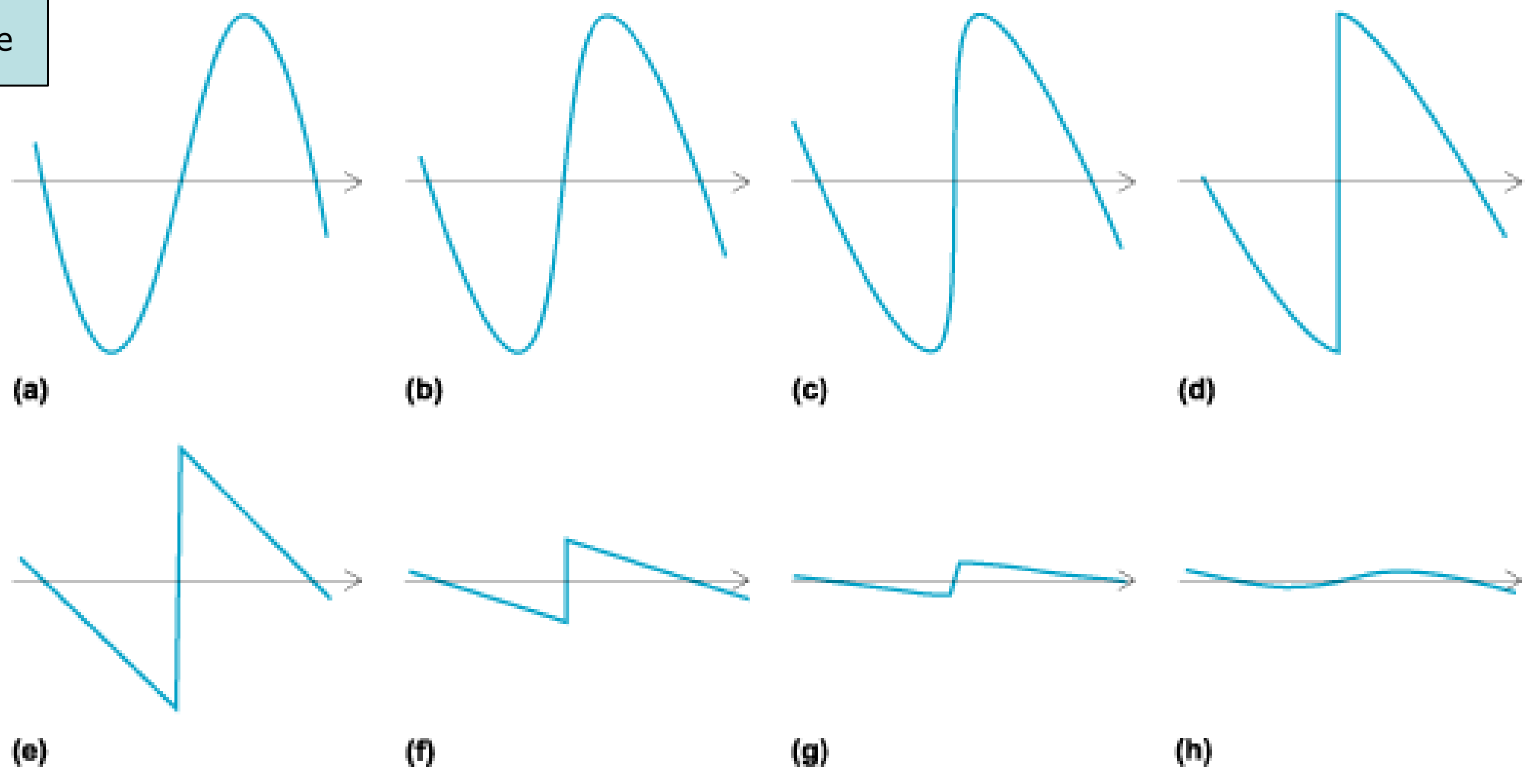
(f) decaying sawtooth,

(g) shock beginning to disperse,

(h) old age.

(After J. A. Shooter et al., Acoustic saturation of spherical waves in water, J. Acous. Soc. Amer., 55:54–62, 1974)

Display
oscilloscope

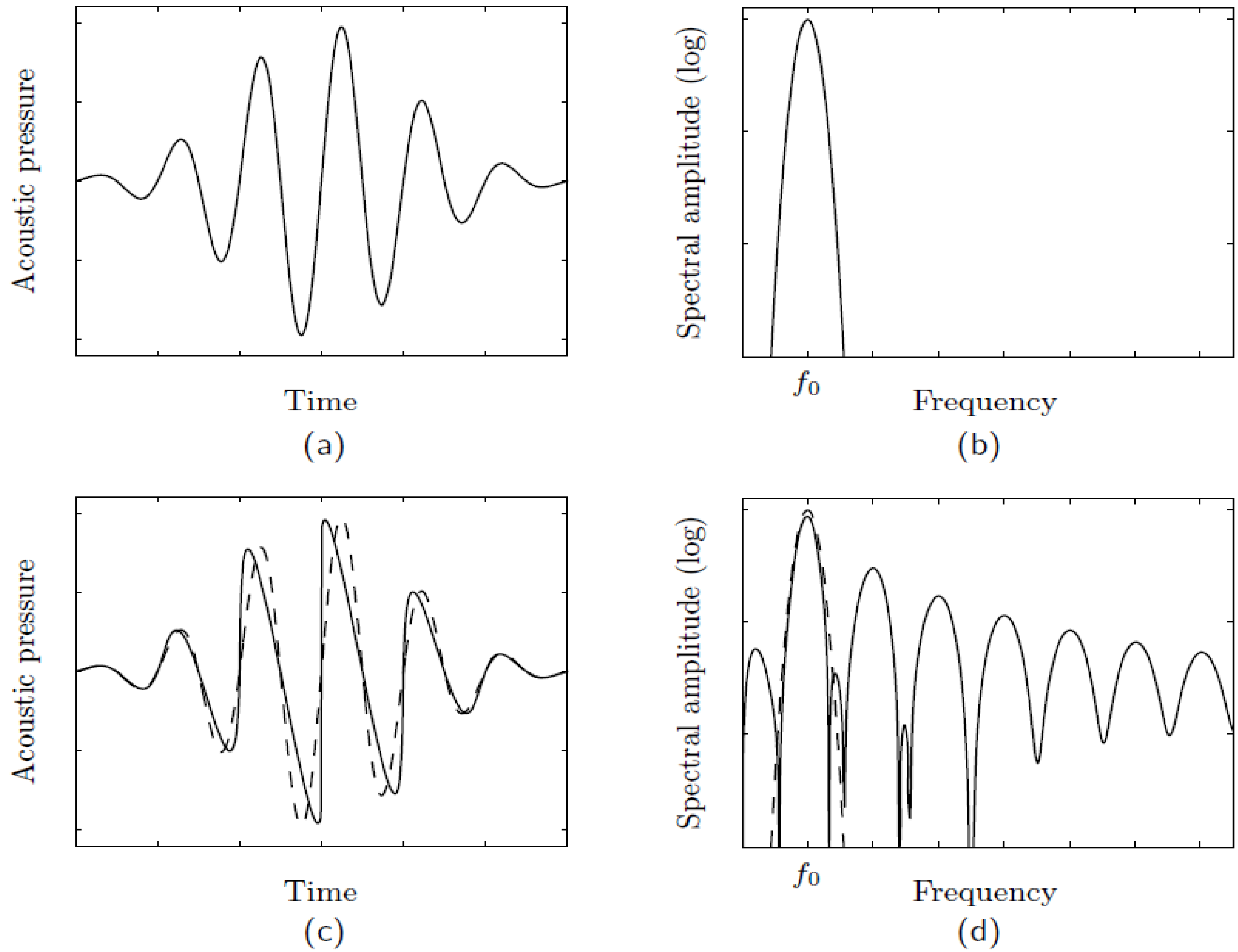


Linear and Nonlinear Propagation

The nonlinear propagation leads to a steepening of the wave form. In the frequency domain this corresponds to the formation of higher harmonic components.

Linear acoustics

Non-linear acoustics



Linear and Nonlinear Propagation

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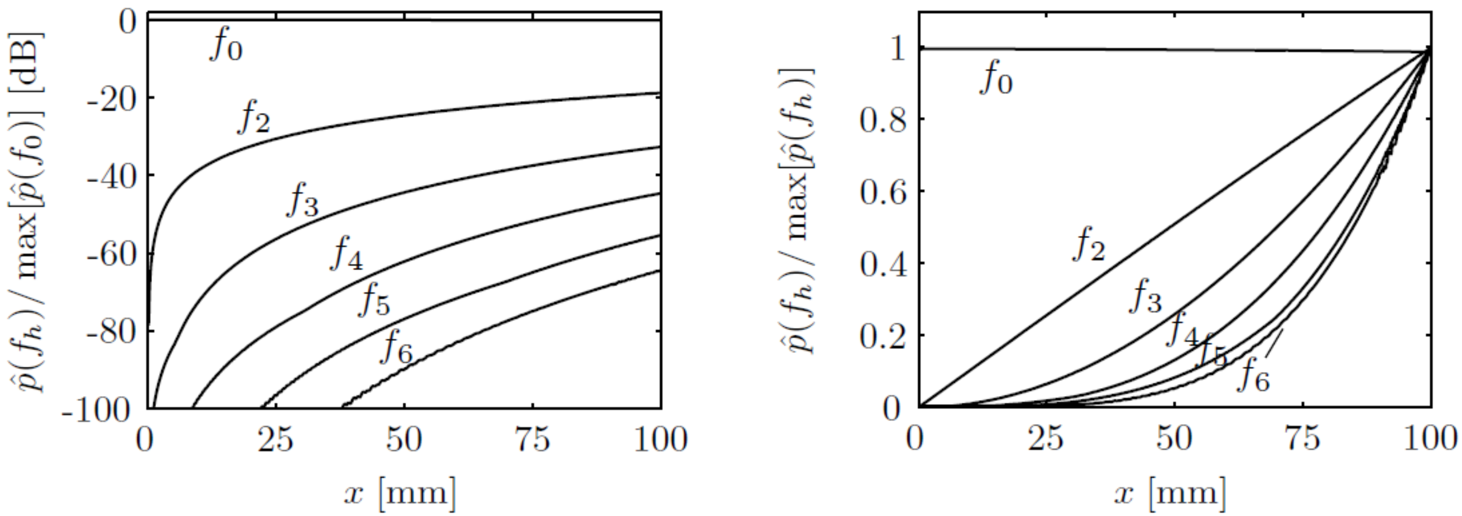

(a) (b)

Nonlinear Wave Equation

Combination of the second order approximations

$$\rho(p) = \rho_0\left[1+\kappa_0(p-p_0)\right],$$
$$\kappa(p) = \kappa_0\left[1+\kappa_0(1-2\beta)(p-p_0)\right]$$

with Hooke's law $\nabla\underline{v}+\rho D_t p=0,$ and Newtons law $\nabla p+\kappa D_t\underline{v}=0,$

leads to the following set of equations

$$\nabla\underline{v}+\left(\rho_0\left[1+\kappa_0(p-p_0)\right]\right)\left(\partial_t+\underline{v}\cdot\nabla\right)p=0$$
$$\nabla p+\left(\kappa_0\left[1+\kappa_0(1-2\beta)(p-p_0)\right]\right)\left(\partial_t+\underline{v}\cdot\nabla\right)\underline{v}=0.$$

Combining the above set of equations and neglecting terms of third order and higher yields the second-order nonlinear wave equation

$$\boxed{\nabla^2 p-\frac{1}{c^2}\partial_t^2 p=-\frac{\beta}{\rho_0 c_0^4}\partial_t^2 p^2}$$

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Nonlinear Wave Equation

Various methods exist to model nonlinear propagation. If the nonlinearity is weak, the additional term may be considered as a contrast source. Next, a solution for the Westervelt equation, which equals

$$\nabla^2 P(\underline{r}) - \frac{1}{c^2}\partial_t^2 P(\underline{r}) = -S^{prime}(\underline{r}) - \frac{\beta}{\rho_0 c_0}\partial_t^2 P^2(\underline{r}),$$

may be obtained by recasting the differential equation into an integral equation, viz.

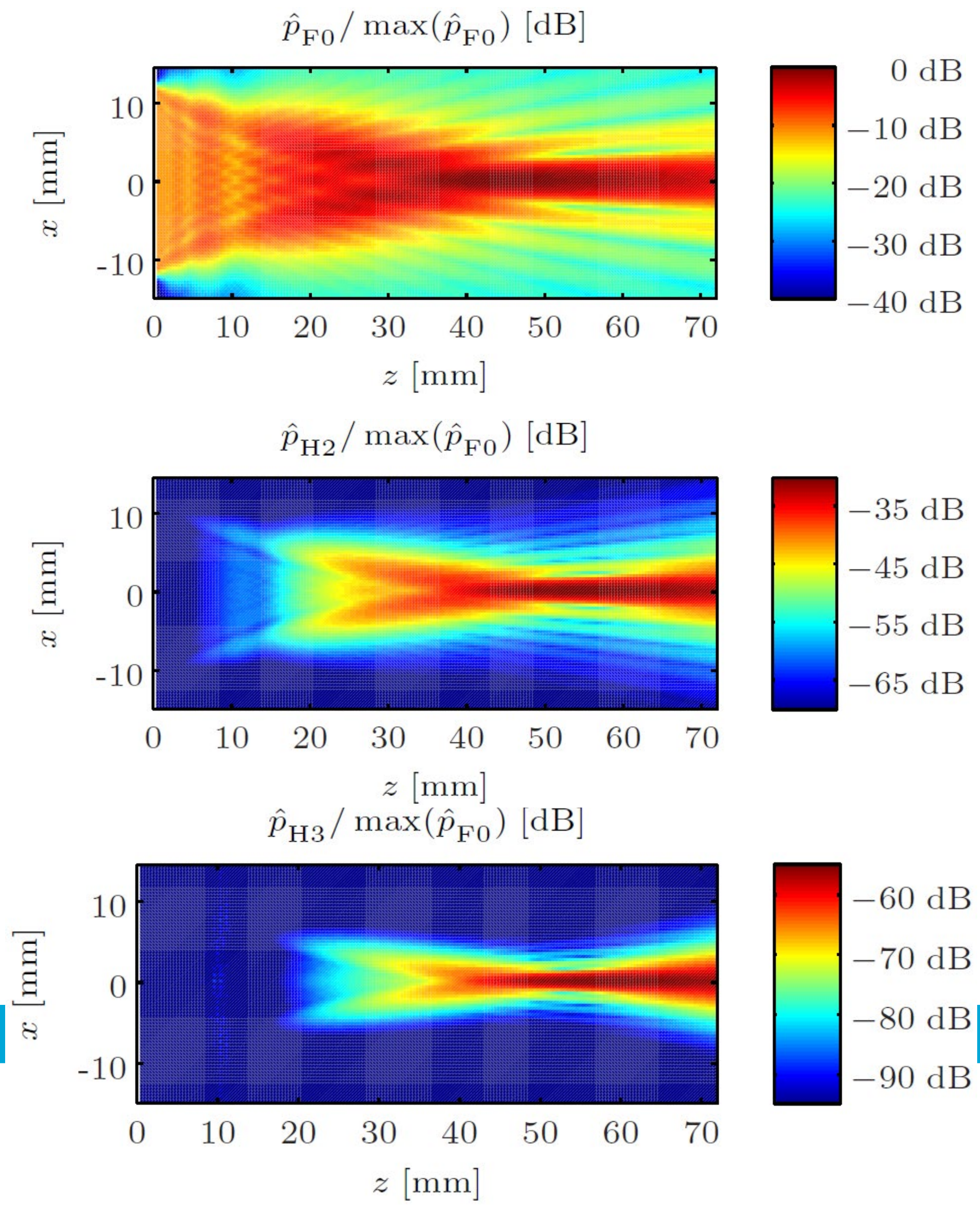
$$P(\underline{r}) = P^{inc}(\underline{r}) - \int_{\underline{r}\in D} G(\underline{r}-\underline{r}')\frac{\beta}{\rho_0 c_0^4}\omega^2\left(P(\underline{r}) *_\omega P(\underline{r})\right) dV.$$

If the nonlinearity is weak, a Neumann scheme is sufficient to solve the integral equation. Note that with each iteration step, one additional harmonic is formed.

Harmonic Imaging

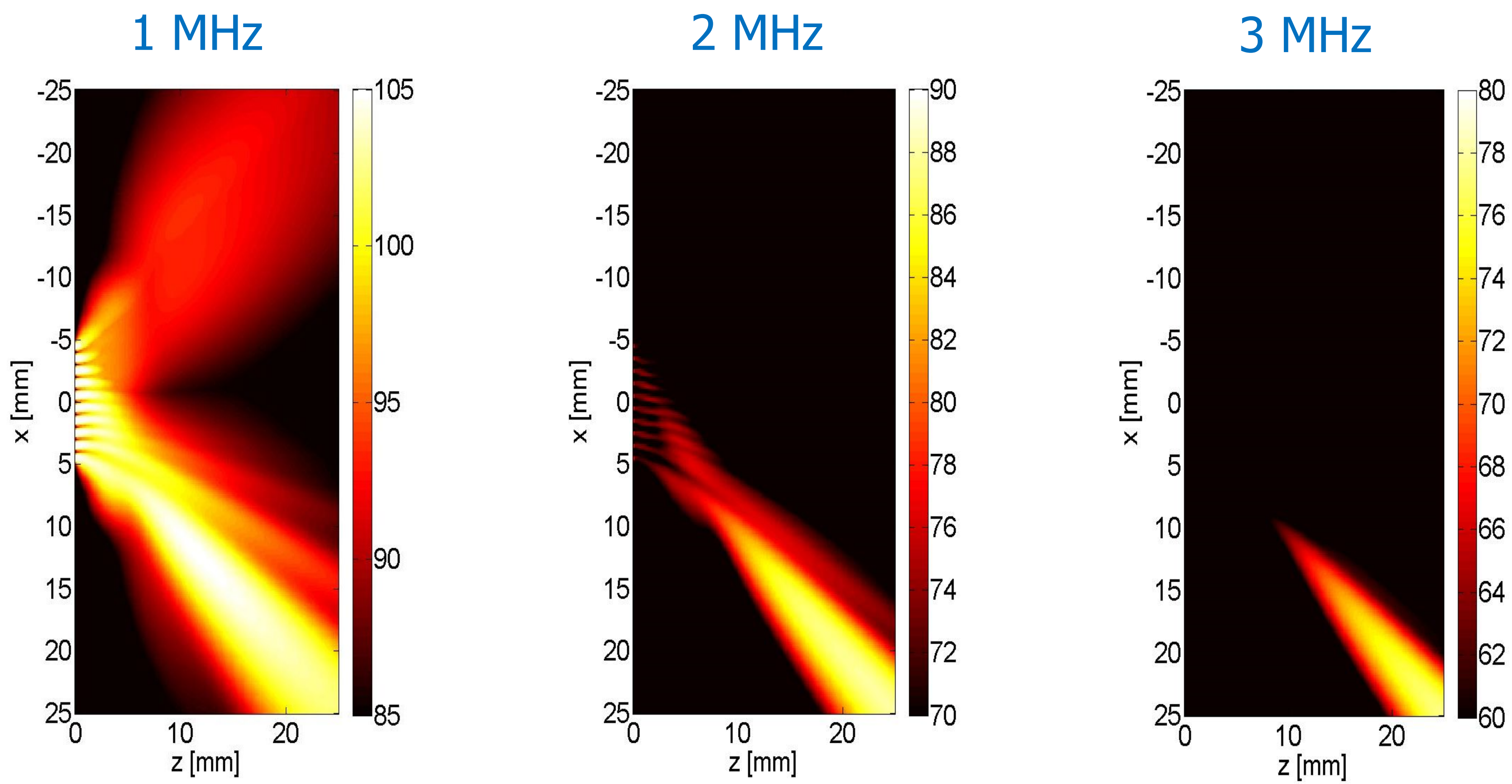
Cardiac Imaging is a well known application for harmonic imaging, as the harmonic components are formed behind the ribs:

- no reflections from the ribs;
- a narrow beam profile.



Harmonic Imaging

The nonlinear propagation can be used to suppress side lobes which are mainly present in fundamental beam.



Hyperthermia with High Intensity Focussed Ultrasound

- HIFU
  - Hyperthermia T = ± 45 °C
  - Ablation T > 60 °C

- Porcine liver

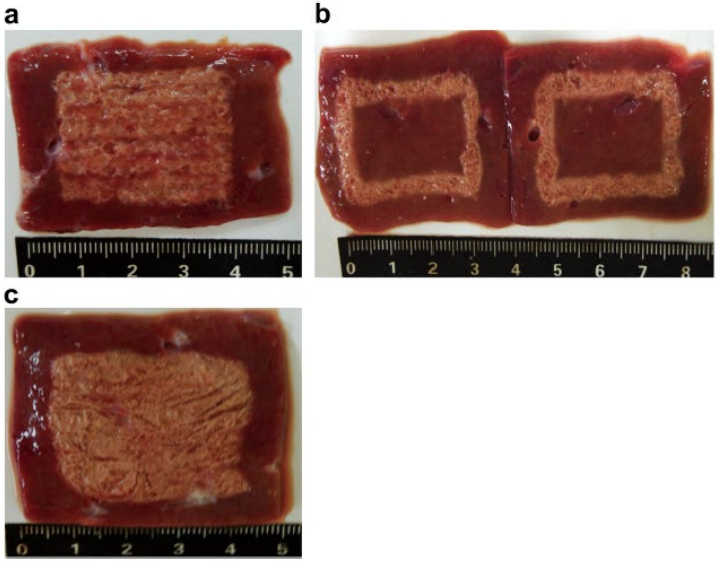


Fig. 6. Gross assessment after peripheral scanning with HIFU. After HIFU ablation according to the peripheral scanning protocol with dual-vessel perfusion, target tissues were cut into 3-mm sections along the height of each sample (*i.e.*, along the *z*-axis). (a) Top surface (*i.e.*, 40-mm depth): with identical generator power (350 W), scanning velocity (4 mm/s) and interval between two adjacent lines (4 mm). (b) Intermediate sections (*i.e.*, 34- and 28-mm depths): at 34-mm depth (left), generator power = 350 W, scanning velocity = 5 mm/s; at 28-mm depth (right), generator power = 350 W, scanning velocity = 6 mm/s. The straight lines scanned only along the periphery of the target region. (c) Undersurface (*i.e.*, 22-mm depth): with identical generator power (300 W), scanning velocity (6 mm/s) and interval between two adjacent lines (4 mm).

Tissue Ablation Accelerated by Peripheral Scanning Mode with High-Intensity Focused Ultrasound: A Study on Isolated Porcine Liver Perfusion Bu, Rui et al. Ultrasound in Medicine and Biology , Volume 39 , Issue 8 , 1410 - 1419

Hyperthermia with High Intensity Focussed Ultrasound

- One may also use ultrasound to measure the temperature

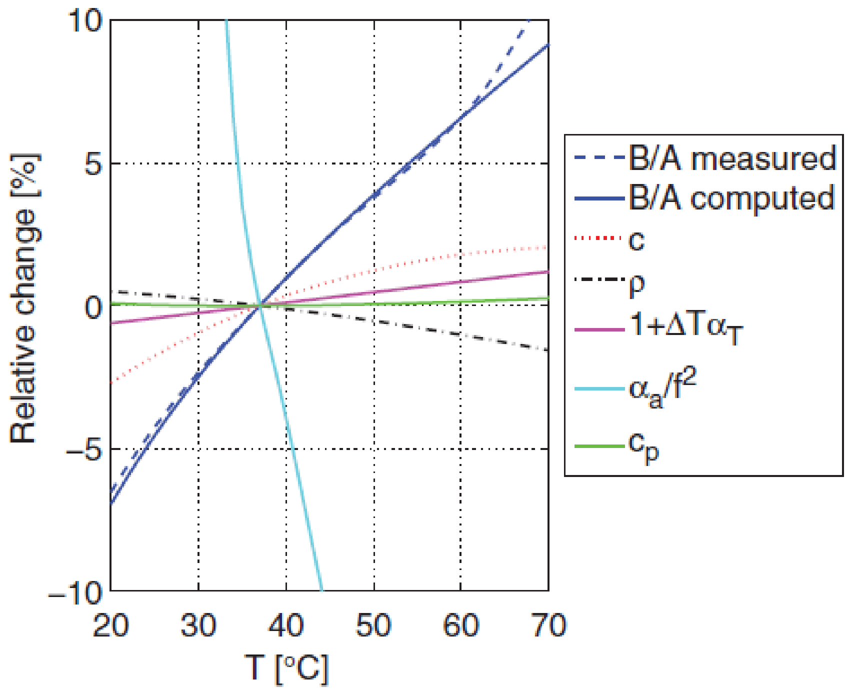


Figure 7. The relative changes of the parameters B/A, c, ρ, $1+\Delta T\alpha_T$, α_a/f^2, and c_p of water. The reference temperature is $T = 37°C$ at ambient pressure.

K.W.A. van Dongen and M.D. Verweij, "A feasibility study for non-invasive thermometry using non-linear ultrasound" Int. J. Hyperthermia (2011)

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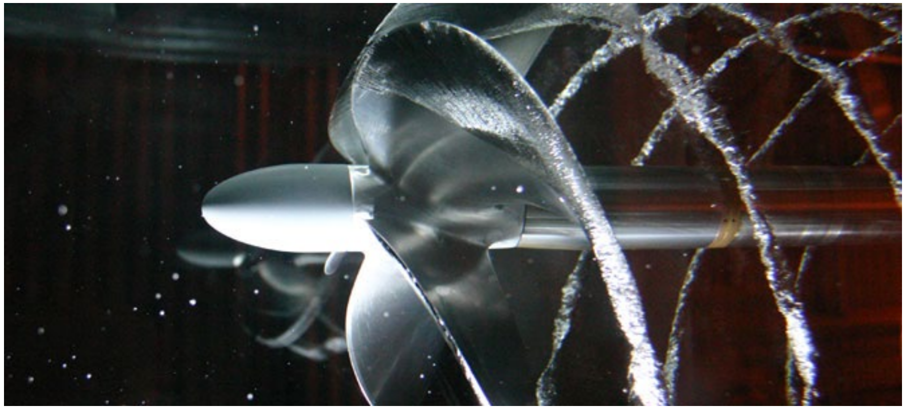
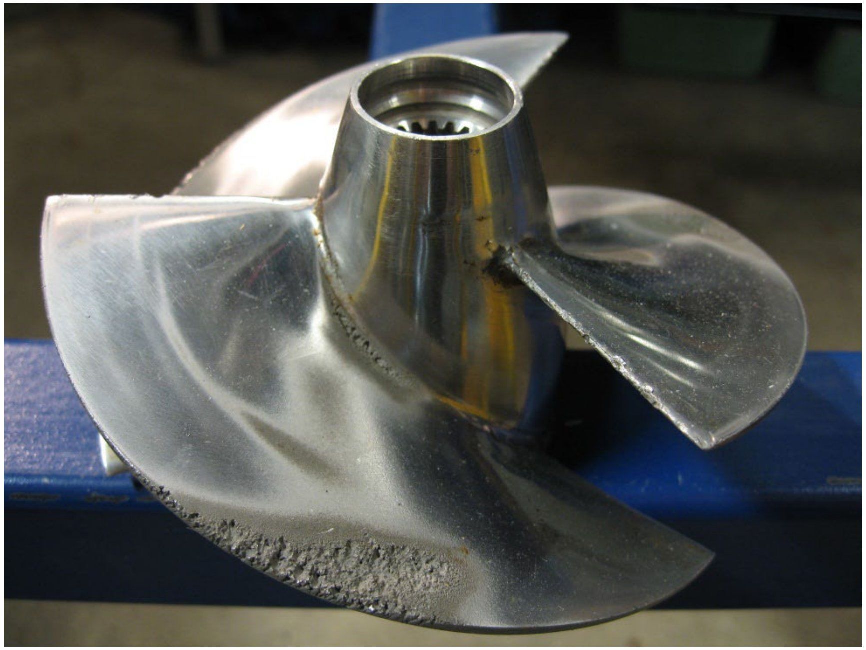
Cavitation

Cavitation refers to the phenomenon of the appearance of holes in liquid. This appearance is due to the stress of tensile forces of some kind. These forces may, in turn, be due to high speed flow, the rapid motion of a solid (propeller blade under water), or to high intensity ultrasound.





www.veempropellers.com
www.britannica.com

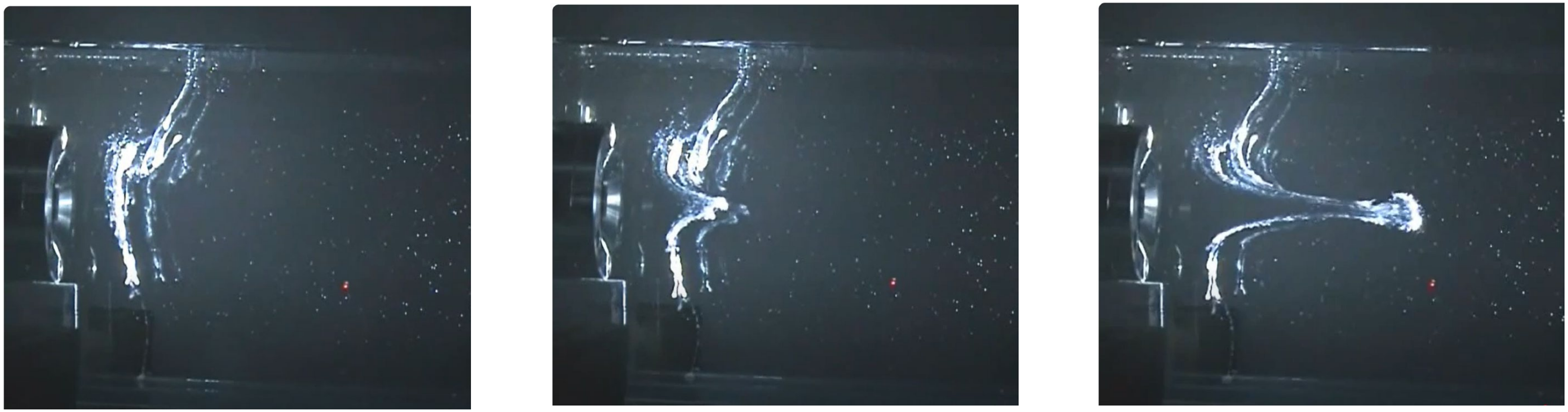
?

- https://youtu.be/ArpclLD4yP8?feature=shared

Acoustic Radiation Force / Acoustic Streaming

- First observed by Faraday in air in 1831.
- Acoustic radiation force is the result of the interaction of the acoustic wave with the medium itself. It follows from conservation of energy and momentum, and only takes place in lossy media.

$$\vec{F} = \frac{2\alpha\vec{I}}{c_p}, \qquad \vec{I} = \langle p\vec{v}\rangle$$



https://youtu.be/ArpclLD4yP8?feature=shared

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