





Part 2: From imaging to inversion





2020







Dussik, https://www.ob-ultrasound.net/dussikbio.html Guasch, *et al.* Full-waveform inversion imaging of the human brain. *npj Digit. Med.* **3**, 28 (2020) **Delft University of Technology**

Conventional Ultrasound Imaging



Heterogeneous Media

- Incident, scattered and total field
- Forward and inverse problem





Acoustic wave equation

Wave equation:

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})} \partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

Helmholtz equation:

$$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c_{bg}^2} p(\vec{x}) = -S^{pr}(\vec{x}) + \omega^2 \underbrace{\left(\frac{1}{c_{bg}^2} - \frac{1}{c^2(\vec{x})}\right)}_{p(\vec{x})} p(\vec{x})$$

$$=\chi(\vec{x})$$

Integral equation:

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') \,\chi(\vec{x}') \,p(\vec{x}') \,\mathsf{d}V(\vec{x}')$$

with
$$G(\vec{x}) = \frac{e^{-ik|\vec{x}|}}{4\pi |\vec{x}|}$$

$$p^{inc} = p + G[p] \rightarrow b = A[x]$$

$$r_n = b - A[x_n]$$

$$x_n = x_{n-1} + \alpha d_n$$

$$d_n = A^{\dagger}[r_n]$$



Breast ultrasound

- Leading cause of death for women (in Western World)
- Screening with mammography is painful, expensive, risks (radiation), limited to elderly women

Alternative: ultrasound?





Breast ultrasound – Siemens Acuson



siemens.com

TU Delft / Erasmus MC Netherlands



Reflectivity versus quantitative imaging





 $R = \frac{c_2 - c_1}{c_2 + c_1}$



Speed of sound Density Compressibility Absorption

Tissue characterisation



Breast ultrasound - an (old) overview





Heterogeneous media – field equations

Hooke's law:
$$\nabla \cdot \underline{v}(\underline{r},t) + \kappa(\underline{r}) \partial_t p(\underline{r},t) = q(\underline{r},t) \Rightarrow \begin{cases} \nabla \cdot \underline{v}(\underline{r},t) + \kappa_0 \partial_t p(\underline{r},t) \\ = q(\underline{r},t) + \left\{\kappa_0 - \kappa(\underline{r})\right\} \partial_t p(\underline{r},t) \end{cases}$$

Newton's law:
$$\nabla p(\underline{r},t) + \rho(\underline{r},t) = \underline{f}(\underline{r},t) \Rightarrow \begin{cases} \nabla p(\underline{r},t) + \rho_0 \partial_t \underline{v}(\underline{r},t) \\ = \underline{f}(\underline{r},t) + \{\rho_0 - \rho(\underline{r})\} \partial_t \underline{v}(\underline{r},t) \end{cases}$$

Combining the above field equations yields the wave equation

$$\nabla^{2} p(\underline{r},t) - \frac{1}{c_{0}^{2}} \partial_{t}^{2} p(\underline{r},t) = -\left\{ \rho_{0} \partial_{t} q(\underline{r},t) - \nabla \cdot \underline{f}(\underline{r},t) \right\}$$
attenuation
$$\int \rho_{0} \left(\kappa_{0} - \kappa(\underline{r}) \right) \partial_{t}^{2} p(\underline{r},t) - \nabla \left\{ \rho_{0} - \mu(\underline{r}) \right\} \partial_{t} \nu(\underline{r},t) \right]$$
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$$\frac{1}{c^{2}(\underline{r})} = \rho_{0} \kappa(\underline{r})$$
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-70

-60

-20

-10

Forward versus Inverse Problem

• Incident, scattered and total field





Acoustic wave equation – forward problem

Wave equation:

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})} \partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

Helmholtz equation:

$$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c_{bg}^2} p(\vec{x}) = -S^{pr}(\vec{x}) + \omega^2 \left(\frac{1}{c_{bg}^2} - \frac{1}{c^2(\vec{x})} \right) p(\vec{x})$$

Radon transform:

$$\Delta t_{\beta}(\gamma) = \int \frac{1}{c(\vec{x})} \, \mathrm{d}\vec{\mathrm{s}}(\vec{x})$$

$$=\chi(\vec{x})$$

Parabolic approximation:

 $p(k_x, k_y, z_0 + \Delta) = p(k_x, k_y, z_0) e^{-ik_z\Delta}$ with $k_z = k_{mean} + (k_x^2 + k_y^2) / 2k_{mean}$

Integral equation:

Born approx.:
$$p \rightarrow p^{inc}$$

') $\chi(\vec{x}') p(\vec{x}') dV(\vec{x}')$ with $G(\vec{x}) = \frac{e^{-ik|\vec{x}|}}{4\pi |\vec{x}|}$

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') \,\chi(\vec{x}') \,p(\vec{x}') \,\mathrm{d}V(\vec{x}')$$



Breast Ultrasound









Breast ultrasound

Synthetic data obtained by solving forward problem for each source.



Inverse radon







Inverse radon

- Bezier Curves: $B(t) = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$
- MUBI system
 - >3MHz
 - $N_{Src/Rec} = N \times 128$





FBB



SoS

Att





(dB/(MHz o 1.8 1.6 1.4 1.2

> 0.8 0.6 0.4

0.2

•••



M. Perez-Liva, et al, "Real-Time Ultrasound Transmission Tomography based on Bézier Curves", Speyer 2017

Parabolic approximation (QT Ultrasound)

- PSolution based on $\mathcal{C}(k_x, k_y, z_0) e^{-ik_z\Delta}$
- Out of plane scattering
- 3D formulation (with parts in 2D?)



Wiskin et al. "3-D Nonlinear Acoustic Inverse Scattering: Algorithm and Quantitative Results", IEEE Trans. UFFC 2017.





Breast ultrasound

Synthetic data obtained by solving forward problem for each source.



Imaging: SAFT / SAR / DAS

Synthetic Aperture Focusing TechniqueSynthetic Aperture RadarDelay and SumSAFT



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SAFT

SoS



-0.05 0 0.05



Saft (MUBI-system)









N.V. Ruiter, et. al. "The USCT reference database", Speyer (2017) N.V. Ruiter, et. al. "Fast reflectivity imaging in 3D using SAFT", Speyer (2017)

Acoustic wave equation based inversion

Wave equation:

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})} \partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

Helmholtz equation:

$$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c_{bg}^2} p(\vec{x}) = -S^{pr}(\vec{x}) + \omega^2 \underbrace{\left(\frac{1}{c_{bg}^2} - \frac{1}{c^2(\vec{x})}\right)}_{=\chi(\vec{x})} p(\vec{x})$$



Inverse problem – Born inversion (linearized inversion)

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') \chi(\vec{x}') p(\vec{x}') dV(\vec{x}')$$

Born Approximation:

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') \, \chi(\vec{x}') \, p^{inc}(\vec{x}') \, \mathrm{d}V(\vec{x}')$$

$$p - p^{inc} = -\omega^2 G^* [\chi p^{inc}]$$
 (b=Ax) Conjuge gradient scheme

$$r_n = p^{sct} - \omega^2 G^* [\chi_n p^{inc}]$$
$$\chi_n = \chi_{n-1} + d_n$$



Linear or Born Inversion

Linear Inversion is an unstable process due to the Born-approximation.





Breast ultrasound



Wave equation

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})} \partial_t^2 p(\vec{x},t)$$

- = $-S^{pr}(\vec{x},t)$ • 2-D inversion (3D too expensive)
- CURE system (Karmanos Cancer Institute)





Figure 7: Recovered models of speed of sound obtained from the 2D in vivo data after using a) FWI at $f_{min} = 500 kHz$ and a constant water starting model, b) bent-ray tomography and FWI with the time-of-flight model as a starting model at c) $f_{min} = 800 kHz$ and d) at $f_{min} = 500 kHz$. The maximum frequency for all FWI tests is 1.75 MHz.

Agudo, et al, "3D imaging of the breast using FWI", Speyer (2017)

Inverse problem – non-linear inversion (full-waveform inversion – FWI – contrast source inversion – CSI)

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') \,\chi(\vec{x}') \,p(\vec{x}') \,\mathrm{d}V(\vec{x}')$$

Born Approximation:

 $p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') \,\chi(\vec{x}') \,p^{inc}(\vec{x}') \,\mathrm{d}V(\vec{x}')$

Full non-linear inversion:

 $p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x} - \vec{x}') w(\vec{x}') dV(\vec{x}') \qquad p^{tot} = p^{inc} - \omega^2 G * w$ $w(\vec{x}') = \chi(\vec{x}') p(\vec{x}') \qquad w = \chi p^{tot}$

$$Err_{n} = \frac{\left\| p - \left(p^{inc} - \omega^{2}G * w_{n} \right) \right\|^{2}}{\left\| p^{inc} \right\|^{2}} + \frac{\left\| w_{n} - \chi_{n-1} \left(p^{inc} - \omega^{2}G * w_{n} \right) \right\|^{2}}{\left\| \chi_{n-1}p^{inc} \right\|^{2}}$$



Non-Linear Inversion

• Non-linear Inversion is a stable process as it uses the full wave equation.





Non-linear inversion

$$p^{tot}(\vec{r}) = p^{inc}(\vec{r}) - \int G(\vec{r} - \vec{r}') \,\omega^2 \chi(\vec{r}') \,p^{tot}(\vec{r}') \,\mathrm{d}V$$

Born Approximation:

$$\hat{p}^{tot}(\vec{r}) = \hat{p}^{inc}(\vec{r}) - \omega^2 \int \hat{G}(\vec{r} - \vec{r}') \,\chi(\vec{r}') \,\hat{p}^{inc}(\vec{r}') \,\mathrm{d}V$$

Full non-linear inversion:

$$\hat{p}^{tot}(\vec{r}) = \hat{p}^{inc}(\vec{r}) - \omega^2 \int \hat{G}(\vec{r} - \vec{r}') \, \hat{w}(\vec{r}') \, \mathrm{d}V$$
$$\hat{w}(\vec{r}') = \chi(\vec{r}') \, \hat{p}^{tot}(\vec{r}')$$

Regularization

- Total Variation
- sparsity

χ

$$p^{tot} = p^{inc} - G * w$$
$$w = \chi p^{tot}$$

$$Err^{(n)} = \left[\frac{\|p^{tot} - p^{inc} + G * w^{(n)}\|}{\|p^{inc}\|} + \frac{\|w^{(n)} - \chi^{(n-1)}(p^{inc} - G * w^{(n)})\|}{\|\chi^{(n-1)}p^{inc}\|}\right] + \left(\begin{array}{c} \\ \end{array} \right)$$





5% noise







All tools





0 x[m]



Iteration 256









Iteration 512

0 x[m] 0.1

-0.1

-0.05

<u>و</u> ٥

0.05

0.1

-0.1

([m]





Frequency versus time domain



Multi-parameter Inversion

In reality, there is a contrasts in *compressibility and density*, besides attenuation.



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U. Taskin et al, "Redatuming of 2-D Breast Ultrasound," IEEE Trans Ultrason Ferroelectr Freq Control 67(1) U. Taskin et al, "Multi-parameter inversion with the aid of particle velocity field reconstruction," JASA 47(6)

Seismic Full Waveform Inversion



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Zhong and Liu, "Time-domain acoustic full-waveform inversion based on dual sensor acquisition system," Journal of Seismic exploration (2019)

Transcranial Ultrasound





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Transcranial Ultrasound















Imaging other body parts

• State of the art: 3-D FWI of breasts and other body parts as well the earth.



3D rendering of the SOS reconstruction of the human knee.



Whole body imaging

• Anatomical correlation of pig abdomen





Wiskin et al., "Whole-Body Imaging Using Low Frequency Transmission Ultrasound, Academic Radiology (2023)

Machine learning for tissue classification

- QT Ultrasound
 - Based on 13 breasts
 - Tissue parameters only (?)
- Problems will occur to apply method to different systems
- Delphinus: Tissue parameters and Texture.





Machine learning for inversion

On synthetic data



On real data



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Zhao *et al* 2023, "Simulation-to-real generalization for deep-learning-based refraction-corrected ultrasound tomography image reconstruction," *Phys. Med. Biol.* **68** (2023)

Acoustics versus *Electromagnetics*, both are waves ...

- ... but with different:
- wavelengths & resolution,
- medium parameters !





P wave velocity model



Resistivity model



Acoustic Inversion



EM Inversion





Can we

• use one reconstruction to enhance the other in an iterative manner? acoustic



electromagnetic

Multi-physics / multi-parameter / joint inversion:

- simultaneous invert for multiple medium parameters,
- regularisation based on structural similarity.



gravity









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Klemm et al., "Experimental and clinical results of breast cancer detection using UWB microwave radar," 2008 IEEE Antennas and Propagation Society International Symposium, 2008.





 $\lambda_{AC} = 15 \text{ mm} << \lambda_{EM} = 225 \text{ mm}$







Joint inversion



Low vs high frequency inversion – Breast US

Dadius = 90 mm	

Radius	= 80 mm
Δx	$= \lambda/6$
RΔφ	$= \lambda/2$
N _f	= 6

		F ₀ =0.5 MHz	F ₀ =2 MHz
		λ=3 mm	λ=0.76 mm
N_{rec}	2-D	330	1.3 · 10 ³
	3-D	17 · 10 ³	2.7 · 10 ⁵
N_{src}	2-D	236	947
	3-D	475	1.8 · 10 ³
N _{unk}	2-D	78 · 10 ³	$1.2 \cdot 10^{6}$
	3-D	8.2 · 10 ⁶	0.5 · 10 ⁹
Mem	2-D	7 GB	455 GB
	3-D	1.4 TB	380 TB

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Mem \approx 4 fields x N_{unk} x N_{src} x N_f x 16

Discussion and conclusion

SAFT

- Echogenicity
- High frequency

Geometric / Ray based methods

- Speed of sound and attenuation
- High frequency
- Many src/rec combinations

Born Inversion / Parabolic methods

- 2D Neglects multiple scattering and phase shifts
- "Speed of sound & attenuation"
- Convergence

Full-Wave Form Inversion

- Taking advantage of multiple scattering and phase shifts
- Speed of sound & attenuation
- 3D Computational heavy -> Low frequencies only

Machine learning

Multi-parameter / Joint Inversion

















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History of ...

- Thales of Miletus 600 BC
- Thales is recognized for breaking from the use of mythology to explain the world and the universe, instead explaining natural objects and phenomena by offering naturalistic theories and hypotheses.





Part 1: History of ultrasound

1793 – Lazarro Spallanzani – Physiologist

Investigated the role of ultrasound for bats.





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https://en.wikipedia.org/wiki/Lazzaro_Spallanzani

1880 – Pierre & Jacques Curie – physicists

Discovery of the piezo-electric effect







1916/1917 – Paul Langevin & Constantin Chilowsky - Physicists

- P. Langevin PhD degree with Pierre Curie (1902)
- Development of the transducer
 - To detect ice berg (Titanic, 1912)
 - WO I: April 23, 1916, sinking U-boat









1928 – S.Y. Sokolov – a Soviet physicist,

- ultrasound to find flaws in metal structures
- \circ through transmission shadow effects in the data
- 1944 Floyd Firestone United States
 - patent for the *Reflectoscope*, first system in which the same transducer both generated the ultrasound waves, and also detected the reflected waves, in the time between transmitted wave pulses.



1942 – Karl Dussik – Neurologist & psychiatric & Friederik Dussik – Physicist

• Imaging the brain



The hyperphonogram was thought to depict the ventricles







1958 – Ian Donald – gynaecologist



 "Investigation of Abdominal Masses by Pulsed Ultrasound", 7 June 1958, The Lancet











Fig. 2. Photograph of complete scanner table installed at Albany Medical Center. The bed is lowered to a horizontal position with the breast positioned through the hole. Fig. 3. Photograph of scanner mechanism. The entire tank rotates to obtain 100 projections. The breast is immersed in water in the inner box.



Glover, "Computerized time-of-flight ultrasonic tomography for breast examination", Ultrasound Med. Biol. 3 (1977)

Fig. 6. Reconstruction for 24 year old asymptomatic person. Internal fibrous structure has velocity



Fig. 2. Patient undergoing ultrasound mammographic examination. Direct water path coupling is used to avoid physical distortion of the breast.



Fig. 5, A. Ultrasound mammogram of a 48year-old woman. A loud echo-producing structure now lies in the center of the breast with an area of decreased reflectivity immediately anterior and posterior to it.

B. Color-coded isodensitometric printout of Figure 5, A.

C. Radiographic mammogram of Figure 5, A.



Fig. 3. (a) Pulse echo, (b) attenuation, and (c) speed-of-sound im-

ages obtained simultaneously in the right breast of a 55-yearold woman. The arrowheads indicate infiltrating ductal carcinoma (1510 m/sec).



The lower bright spot represents the posterior portions of the subareolar tissues (1516 m/sec). The remainder of the breast plane is predominantly fatty tissue (1430 m/sec).

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Carson, Meyer, Scherzinger, Oughton, "Breast Imaging in Coronal Planes with Simultaneous Pulse Echo and Transmission Ultrasound", Science (1981)

1986 – Real-time orthogonal mode scanning of the heart.

J.E. Snyder, J. Kisslo, O.T. von Ramm



1991 – First hand held transducer for 3-D real-time ultrasound imaging

S.W. Smit, H.G. Pavy, O.T. von Raman

32 transmit channels32 receive channels17 x 17 elements



Fig. 1. Pyramidal scan format of 3-D ultrasound imaging system showing phased array transducer (front view), C-scan image plane and projection display points.





1991 – First hand held transducer for 3-D real-time ultrasound imaging



4.5 mm x 4.5 mm 120 μm x 120 μm













Brief history of seismic

- -JUD Seismic community is The seismic ales alead of us two decades area of the two decades area of two decades are 1920s
- 1960s
- 1970s
- 1980s •

2017

2010100,000

seismic survey in 1940's (wikipedia)

Medical vs seismic "survey"

Medical	Seismic
F = 0.1 - 100 MHz	F = 0.1 - 1000 Hz
Real time	Days – weeks – months
Beam steering	Point sources / receivers
Geometric based	Wave based
1-128 channels	>10 ⁶ channels
2D (3D)	3D
Price system 5-200 k€ Price scan 0.1-0.2 k€	? 5-700 k€ /km ² (land)

