Correspondence

Discussion of One-D Piezoelectric Transducer Models with Loss

Glenn Whitworth, *Member, IEEE*

***Abstract*—Two, 1-dimensional piezoelectric transducer models are presented that use complex numbers to represent mechanical and dielectric loss. Exact numerical agreement was achieved by avoiding a number of small errors in the literature. A correction to Kino's simplified Mason parallel equivalent circuit is also given. Efficiency is discussed.**

I. Introduction

The simplified Mason equivalent circuits described by Kino [1] continue to be accepted and used to provide first-order approximation of piezoelectric transducers [2]. Other widely recognized and more accurate 1-D piezoelectric transducer models include the (full) Mason model [3]–[5], the KLM model [6], [7] implemented using the T-matrix formalism [8], [9], and models based on a system of equations deduced from Coursant [10], [11].

Section II of this article describes two computer models based on the literature and gives the equations required for them to be in exact mutual agreement. A number of small errors in the literature are avoided. In Section III, one of these models is then numerically compared with Kino's simplified Mason equivalent circuits, and a resulting disagreement is resolved. Finally, efficiency is discussed. Each of these models was programed in Mathcad7 Professional (MathSoft, Inc.) and run on an Intel Pentium-based computer (Digital Equipment Corp.).

II. Piezoelectric Transducer Models with Loss

Two piezoelectric transducer models are described in this section starting with equations that are common to both. The models represent a transducer with a single piezoelectric layer and the option of acoustic matching layers. Bhatia [12] has shown that acoustic loss can be represented using real frequency and complex phase velocity. Mechanical and dielectric losses will be included in the piezoelectric layer through the use of a complex stiffened wave speed v_a and complex clamped capacitance C_0, respectively. The complex stiffened wave speed v_a may be represented by

$$v_a \approx v_{re} \cdot \left(1 - \frac{j}{2Q_m}\right)^{-1} \approx v_{re} \cdot \left(1 + \frac{j}{2Q_m}\right) \qquad (1)$$

where Q_m is the mechanical quality factor and v_{re} is the real part of v_a. The propagation constant γ is given by

$$\gamma = j \cdot \frac{2\pi f}{v_a}. \qquad (2)$$

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The author is an independent consultant in Marlborough, NH 03455 (e-mail: whitworth@ieee.org).

Eq. (1) and (2) can be easily derived from [9, Table I], and the relationship between attenuation α and Q_m is given by $Q_m = \pi f / \alpha v_{re}$. The stiffened mechanical impedance Z_c is given by

$$Z_c = \rho v_a A \qquad (3)$$

where ρ is the piezoelectric's density and A is its area. When mechanical loss is present, it is evident that Z_c and γ are both complex. Similar equations would be used to represent the optional acoustic matching layers with loss. The clamped capacitance C_0 may be represented by

$$C_0 = \frac{\varepsilon_r \varepsilon_0 A}{d} \cdot [1 - j\tan(\delta)] \qquad (4)$$

where ε_r is the relative clamped permittivity, ε_0 is the permittivity of free space, d is the thickness, and $\tan\delta$ represents the loss tangent. Eq. (1) and (4) provide convenient representations of loss, but they may be modified to better model the loss of specific materials [12]. Two models that will use these equations will now be described.

A. Model 1

Model 1 is based on the matrix from Dion *et al.* [11, eq. (1)] or equivalently, Rhyne [5, eq. (1)]. The 3×3 matrix was converted into 2- $\times$ 2-form by basic algebraic manipulation. A resulting matrix M_{total}, which represents the transfer function of the transducer, is given by

$$M_{total} = M1^{-1} \cdot M2 \cdot M_{front} \qquad (5)$$

where

$$M1 = \begin{bmatrix} F & G^2 - H \cdot F \\ E + Z_{backing} & G^2 - H \cdot (E + Z_{backing}) \end{bmatrix}, \qquad (6a)$$

$$M2 = \begin{bmatrix} G & -G \cdot (F - E) \\ 0 & -G \cdot (E + Z_{backing} - F) \end{bmatrix}, \qquad (6b)$$

$$E = Z_c / \tanh(\gamma\ d), \quad F = Z_c / \sinh(\gamma\ d), \qquad (7a)$$

$$G = h / (j \cdot 2\pi f), \text{ and } H = 1/(j \cdot 2\pi f \cdot C_0). \qquad (7b)$$

$Z_{backing}$ is the mechanical impedance of the backing and M_{front} may be used to represent the option of acoustic matching layers. Additional expressions necessary for using the T-matrix formalism to calculate various results are presented by Oakley [9]. Model 1 was tested at a variety of frequencies and acoustic loads, with and without mechanical and dielectric loss. It was found to have a singularity at the parallel resonance frequency for the (physically impossible) case of no internal loss. Except near this point, the transducer input impedance was found to be in exact agreement with [11, eq. (6)]. (The word "exact" will be used in this article to mean at least 15 significant digits of agreement.) With loss, the agreement at and near the parallel resonance frequency was extremely close and so also supports the view that this model is analytically correct.

B. Model 2

Model 2 is a KLM model. (For a complete description of the KLM model with similar notation see Oakley [9] or Zipparo *et al.* [7].) The expression used for the electromechanical transformer turns ratio was

$$\Phi = \frac{h}{2 f_p Z_c} \cdot \mathrm{sinc}\left(\frac{f}{2 f_p}\right) \tag{8a}$$

where h is the stress piezoelectric constant. The directionality of the ratio Φ is such that the appropriate matrix for use in the T-matrix formalism is

$$\begin{bmatrix} \Phi & 0 \\ 0 & \Phi^{-1} \end{bmatrix}. \tag{8b}$$

The acoustically unloaded, fundamental, parallel resonance frequency f_p of the piezoelectric is given by

$$f_p = \frac{v_a}{2d}, \tag{9}$$

and so it is evident that f_p must be complex when mechanical loss is present. The following series capacitance C' was used:

$$C' = \frac{-2 f_p Z_c}{h^2 \cdot \mathrm{sinc}(f/f_p)}. \tag{10}$$

At all tested frequencies including f_p, with and without mechanical and dielectric loss, Model 2 was found to produce numerical results that were in exact agreement with [11, eq. (6)]. Model 2 calculates somewhat faster than Model 1 and also has no singularity at f_p. Model 1 was easier to debug and will also better facilitate discussion of loss mechanisms.

The validated KLM implementation, Model 2, will next be compared with the simplified Mason equivalent circuits. These comparisons require a relationship between h and the thickness coupling constant k_T. The following relation was used:

$$h = k_T \sqrt{\frac{2 f_p Z_c}{C_0}}. \tag{11}$$

III. The Simplified Mason Equivalent Circuits

Two simplified Mason equivalent circuits are given by Kino [1] and restated by Mills and Smith [2]. One is a series equivalent circuit, and the other is a parallel equivalent circuit; they are approximate solutions valid only near the parallel and series resonances, respectively. Kino [1] and Mills and Smith [2] disagree as to which circuit should be called series and which parallel. Neither circuit includes mechanical or dielectric loss. The series equivalent circuit {[1, Fig. 1.4.9c] or [2, Fig. 1 (right)]} was found to be in exact agreement with Model 2 at f_p. But the parallel equivalent circuit {[1, Fig. 1.4.11b] or [2, Fig. 1 (left)]} showed significant disagreement with Model 2 when compared at f_s.

A simple example will now be chosen to demonstrate the problem. The example is a piezoelectric plate radiating from both faces into media with mechanical impedances Z_1 and Z_2. The properties are arbitrarily chosen to be C_0 = 2.849 nF, d = 1 mm, A = 5.067×10^{-4} m^2, v_a = 4600 m/s, k_T = 0.481, Z_c/A = 34.5 MRayl, Z_1/A = 4 MRayl, and Z_2/A = 8 MRayl.

A comparison was made with Model 2 at the series resonance frequency f_s. The well-known relationship among f_s, f_p, and k_T, given by [7, eq. (5)], was used to calculate f_s. Approximate calculated values for f_p and f_s are 2.3 and 2.06 MHz, respectively.

At f_s, the parallel equivalent circuit predicts an electrical conductance of 31.252 mS, but Model 2 predicts a conductance of 34.870 mS. This error of more than 10% can be almost eliminated by changing the resistance of the equivalent circuit's motional branch to read

$$R = \frac{\pi \cdot (Z_1 + Z_2)}{4 k_T^2 \omega_p C_0 Z_c}. \tag{12}$$

The only change made here is that ω_s has been replaced with ω_p (notwithstanding the fact that we are interested in the series resonance). Eq. (12) predicts a conductance at f_s of 34.867 mS. Disagreement with Model 2 is now less than 0.01%. The motional resistance (if expressed in terms of h) is now also in analytical agreement with Gooberman [4, Fig. 3.12]. The parallel equivalent circuit remains an approximation, however, and is still only valid near the series resonance.

IV. Discussion and Conclusions

Kino [1] and Gooberman [4] each present a simplified Mason parallel equivalent circuit, but they disagree. Kino's version has propagated in the literature and has been used to calculate impedance at the series resonance [2]. Eq. (12) was obtained by changing a single subscript in Kino's motional resistance term. When applied to a typical example, this modification was shown to reduce error by a factor of over 1000 at the series resonance. This modification is also analytically supported by Gooberman's work [4], which agrees that the motional resistance term is correctly given by (12).

To facilitate the process of cross-checking and debugging new simulation programs, two models were presented in Section II that have been tested to be in exact numerical agreement. Mechanical loss was introduced through the complex stiffened wave speed v_a, while using a real frequency f. Agreement between models was achieved when f_p, Z_c, and γ, which are all functions of v_a, were all treated as complex. To represent dielectric loss, C_0 was treated as complex. With reference to (7) of Model 1, it is evident that mechanical loss is introduced through terms E and F, and dielectric loss of the clamped capacitance is introduced through term H. An additional influence on efficiency is introduced through the piezoelectric term G if h is complex. (Efficiency is being defined as the ratio of output acoustic power to the transducer's input electrical power.)

There is a final pitfall to be avoided when applying these models. Simulations support the assertion that, if h is real, then the predicted efficiency will never exceed 100% with any combination of positive values for tan δ and Q_m. But, in the literature, it is not uncommon for transducer models to be expressed directly in terms of k_T rather than h, and so k_T is often approximated by a real number. To avoid unintended results, one should be careful when using approximate or hypothetical material properties. For example, simulations show that, if a real k_T is used to model a hypothetical transducer that has dielectric loss but negligible mechanical loss, then a nonsensical prediction of over 100% efficiency will result.

Appendix A

This is a list of small disagreements with otherwise excellent articles.

- Eq. (1) and (2) are omitted by [7] and disagree with [9, eq. (8)].
- Eq. (3) disagrees with [9, eq. (9)].
- For (8a) and (8b), [1], [7], and [9] use diagrams to define the ratio Φ. The diagrams agree, but the Φ of [7] is the reciprocal of [1] and [9].
- Eq. (9) disagrees with [7] and [9], who both treat f_p as real.
- Eq. (10) disagrees with [7, eq. (2)], which has a wrong sign, and with [9, eq. (6)], which has a misplaced sinc().
- Eq. (11) disagrees with an unnumbered equation in [5, p. 1137].
- Eq. (12) disagrees with [1, Fig. 1.4.11b] and disagrees with [2, Fig. 1 (left)].

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