PART 2:

From imaging to multi-parameter inversion

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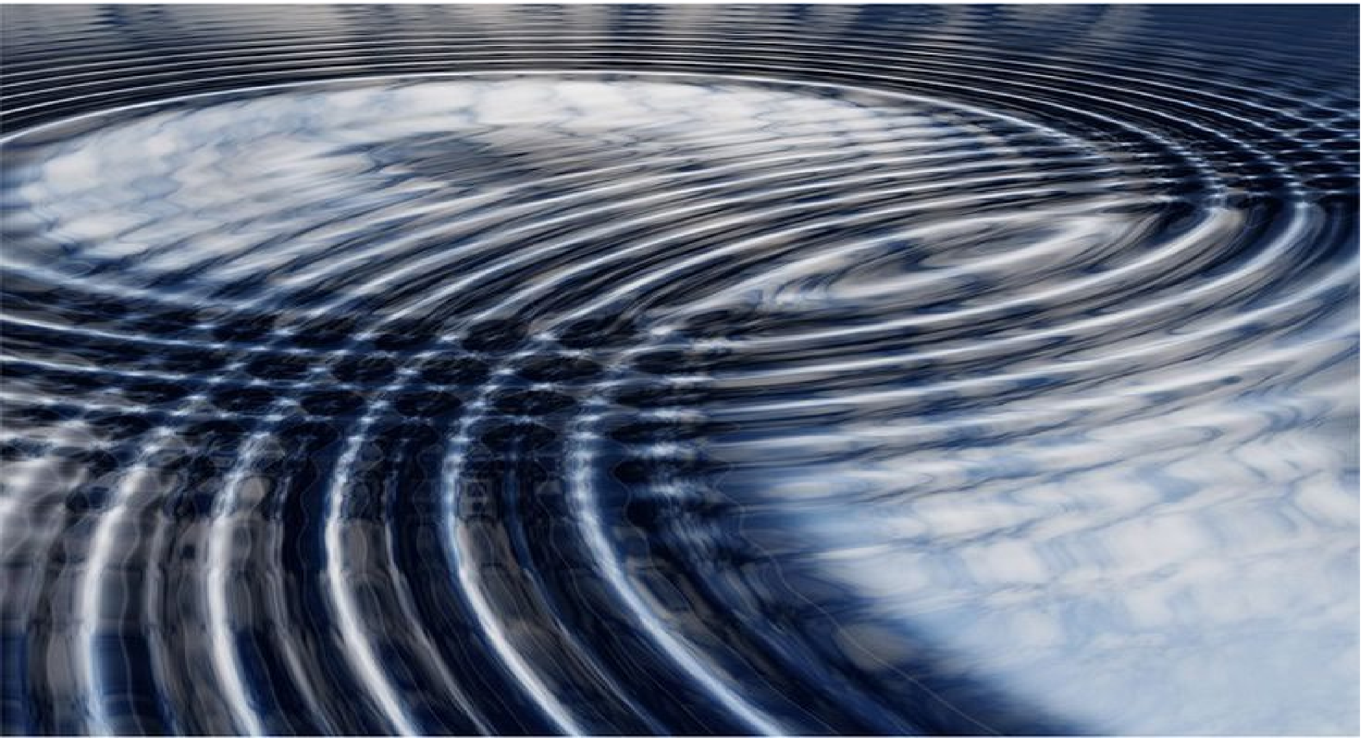

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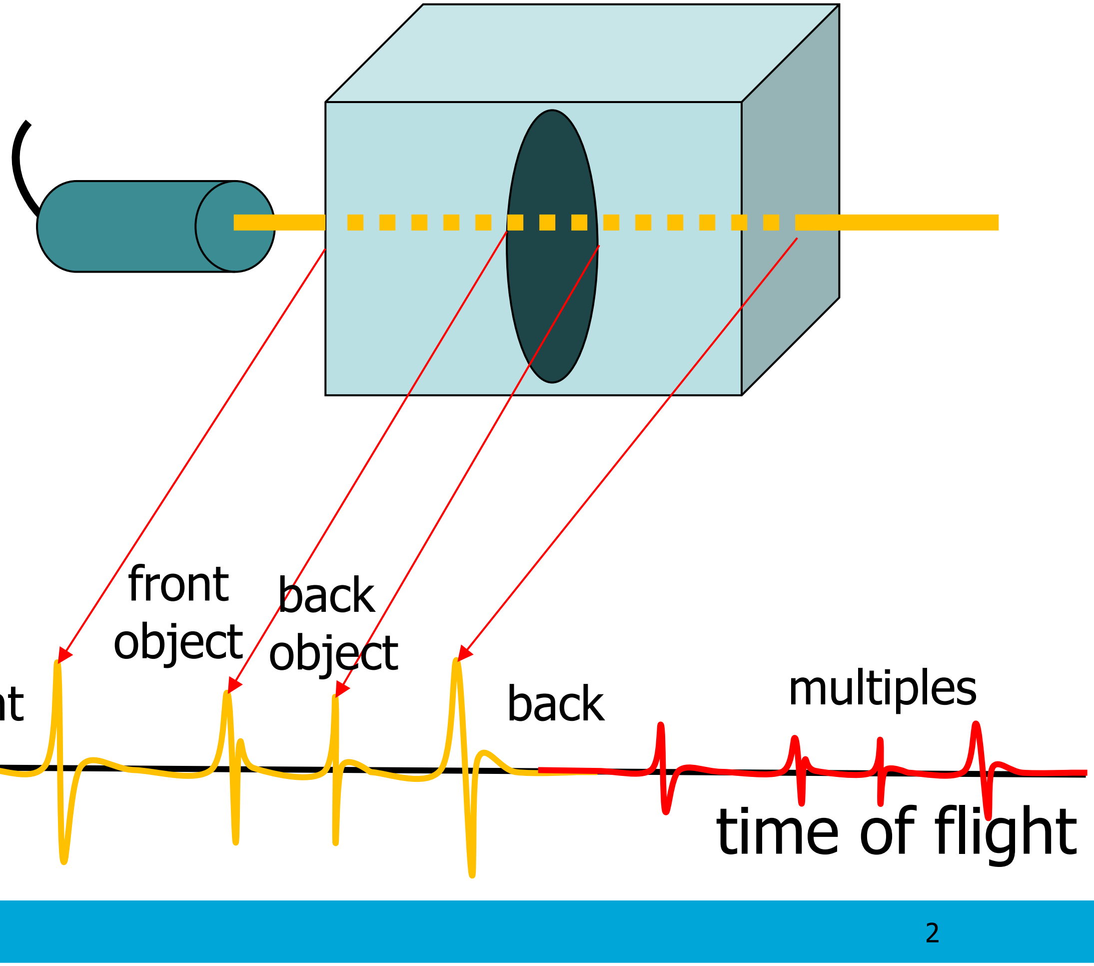
Ultrasound scan



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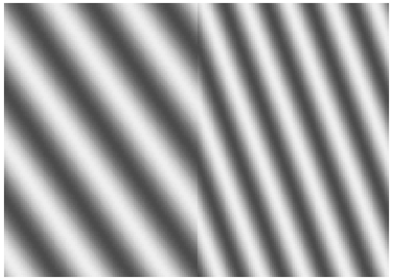
Breast ultrasound – Siemens Acuson

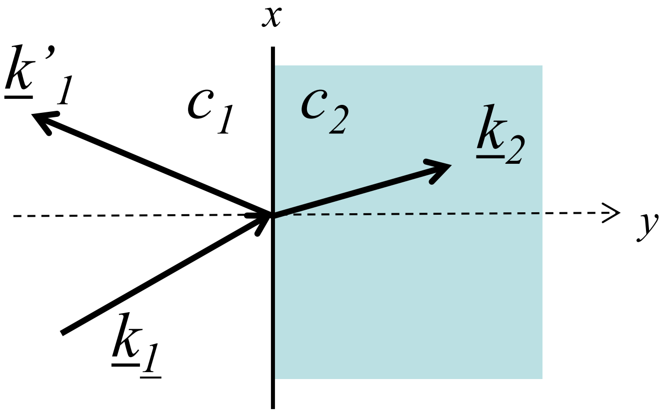
siemens.com

TU Delft / Erasmus MC Netherlands

Two Media - Boundary Conditions



Consider a plane wave traveling in the $(x, y)-$plane
with wave vector $\underline{k}_1$, with velocity c_1: $\hat{p}_1(\underline{r},\omega) = I(\omega)e^{-i\underline{k}_1\cdot\underline{r}}$



If the field meets a boundary between two media
with different speed of sound,
part of the field will be reflected: $\hat{p}'_1(\underline{r},\omega) = R(\omega)e^{-i\omega\underline{s}'_1\cdot\underline{r}}$,
and part of field will be refracted: $\hat{p}_2(\underline{r},\omega) = T(\omega)e^{-i\omega\underline{s}_2\cdot\underline{r}}$.

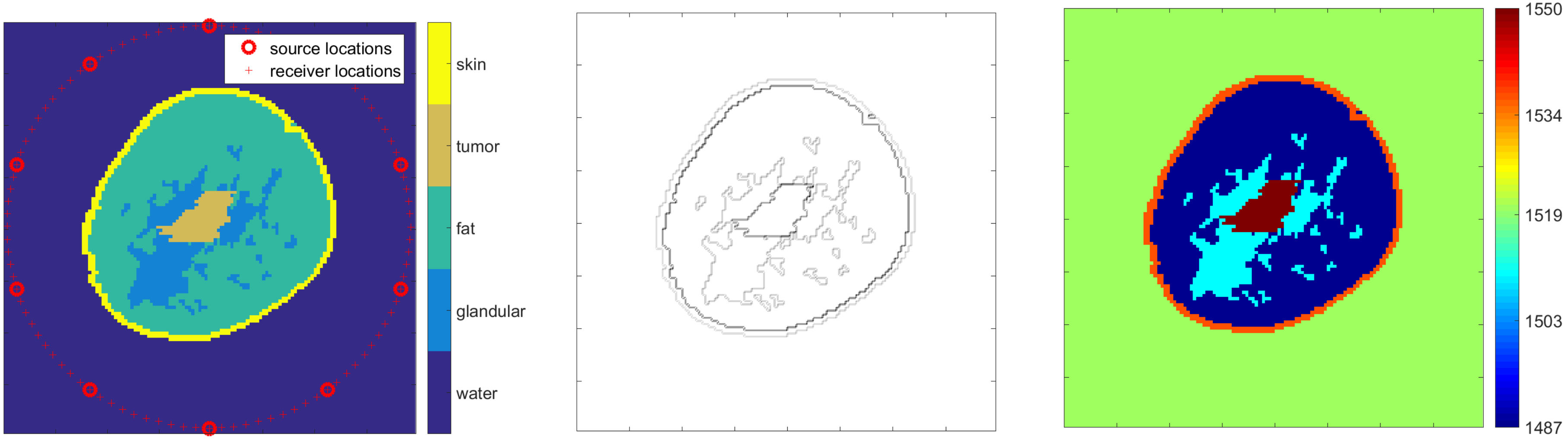
At $y = 0$, the following boundary conditions apply:
1) continuity of pressure: $\boxed{\hat{p}_1 + \hat{p}'_1 = \hat{p}_2}$;
2) continuity of normal component of the particle velocity: $\boxed{v_1^{\perp} + v'^{\perp}_1 = v_2^{\perp}}$.

$\rho_0 c \equiv Z$

$$R = \frac{Z_2\cos\vartheta_1 - Z_1\cos\vartheta_2}{Z_2\cos\vartheta_1 + Z_1\cos\vartheta_2}$$

$$T = \frac{2Z_2\cos\vartheta_1}{Z_2\cos\vartheta_1 + Z_1\cos\vartheta_2}$$

Reflectivity versus quantitative imaging



$$R = \frac{c_2 - c_1}{c_2 + c_1}$$

Speed of sound
Density
Compressibility
Absorption

Tissue characterisation

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Heterogeneous media – field equations

Hooke's law: $\nabla\cdot\underline{v}(\underline{r},t)+\kappa(\underline{r})\partial_t p(\underline{r},t)=q(\underline{r},t)\Rightarrow$

$$\nabla\cdot\underline{v}(\underline{r},t)+\kappa_0\partial_t p(\underline{r},t)=q(\underline{r},t)+\{\kappa_0-\kappa(\underline{r})\}\partial_t p(\underline{r},t)$$

Newton's law: $\nabla p(\underline{r},t)+\rho(\underline{r})\partial_t\underline{v}(\underline{r},t)=\underline{f}(\underline{r},t)\Rightarrow$

$$\nabla p(\underline{r},t)+\rho_0\partial_t\underline{v}(\underline{r},t)=\underline{f}(\underline{r},t)+\{\rho_0-\rho(\underline{r})\}\partial_t\underline{v}(\underline{r},t)$$

Combining the above field equations yields the wave equation

$$\nabla^2 p(\underline{r},t)-\frac{1}{c_0^2}\partial_t^2 p(\underline{r},t)=-\underbrace{\left\{\rho_0\partial_t q(\underline{r},t)-\nabla\cdot\underline{f}(\underline{r},t)\right\}}_{S_{pr}(\underline{r},t)}$$

$$-\underbrace{\left\{\rho_0\left(\kappa_0-\kappa(\underline{r})\right)\partial_t^2 p(\underline{r},t)-\cancel{\nabla[\{\rho_0-\rho(\underline{r})\}\partial_t\underline{v}(\underline{r},t)]}\right\}}_{S_{cs}(\underline{r},t)}$$

$\boxed{\text{attenuation}}$

$$\boxed{\frac{1}{c^2(\underline{r})}=\rho_0\kappa(\underline{r})}$$

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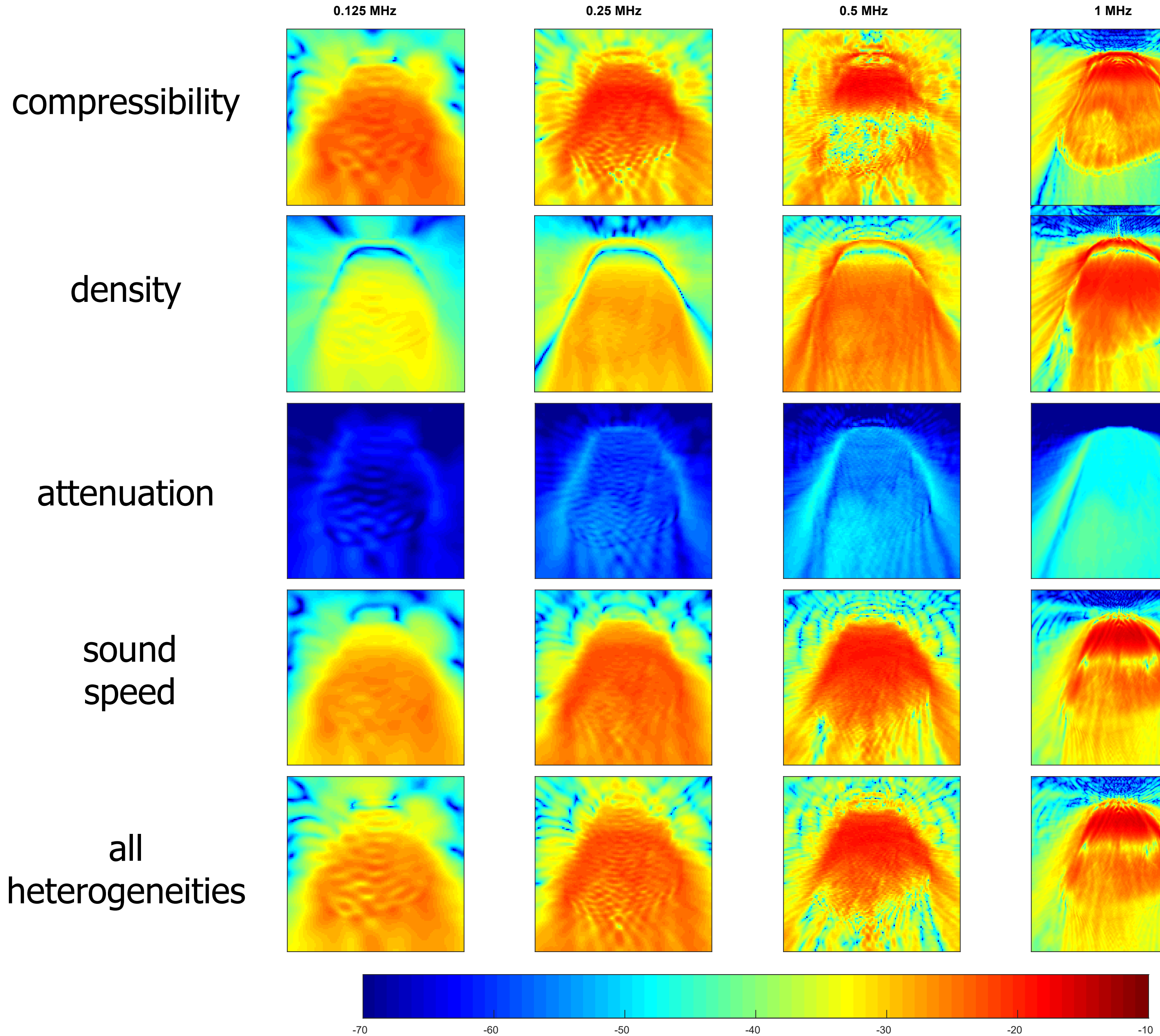


0.125 MHz
0.25 MHz
0.5 MHz
1 MHz
compressibility
density
attenuation
sound speed
all heterogeneities
-70
-60
-50
-40
-30
-20
-10

Acoustic wave equation – forward problem

Wave equation: $$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})}\partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

Helmholtz equation: $$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c_{bg}^2} p(\vec{x}) = -S^{pr}(\vec{x}) + \omega^2 \underbrace{\left(\frac{1}{c_{bg}^2} - \frac{1}{c^2(\vec{x})}\right)}_{=\chi(\vec{x})} p(\vec{x})$$

Radon transform: $$\Delta t_\beta(\gamma) = \int \frac{1}{c(\vec{x})}\, \mathrm{d}\vec{s}(\vec{x})$$

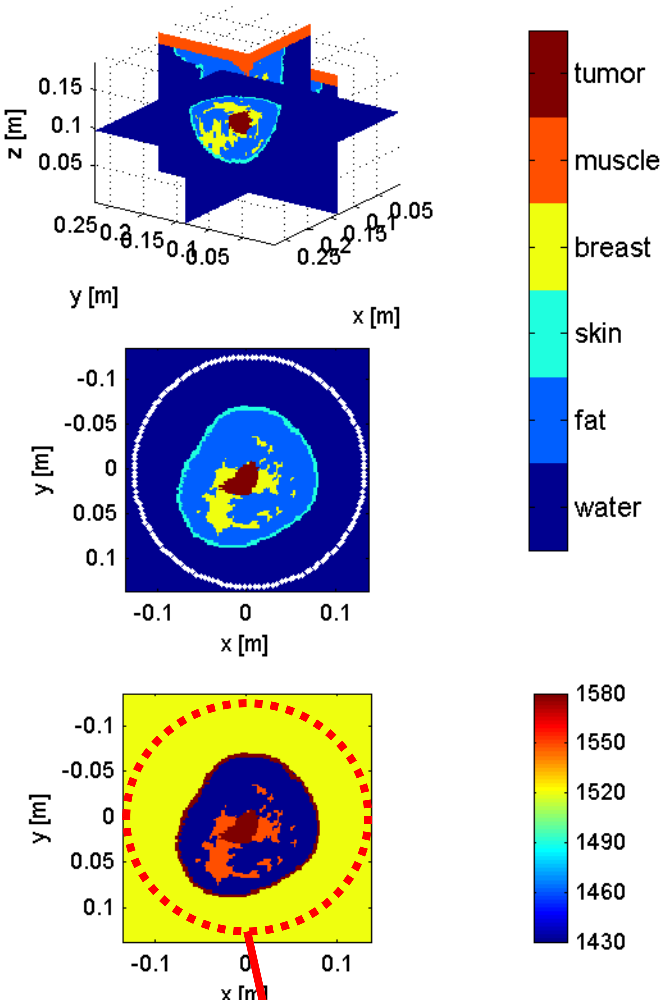
Parabolic approximation:
$$p(k_x,k_y,z_0+\Delta) = p(k_x,k_y,z_0)\, e^{-ik_z\Delta} \quad \text{with} \quad k_z = k_{mean} + (k_x^2 + k_y^2)/2k_{mean}$$

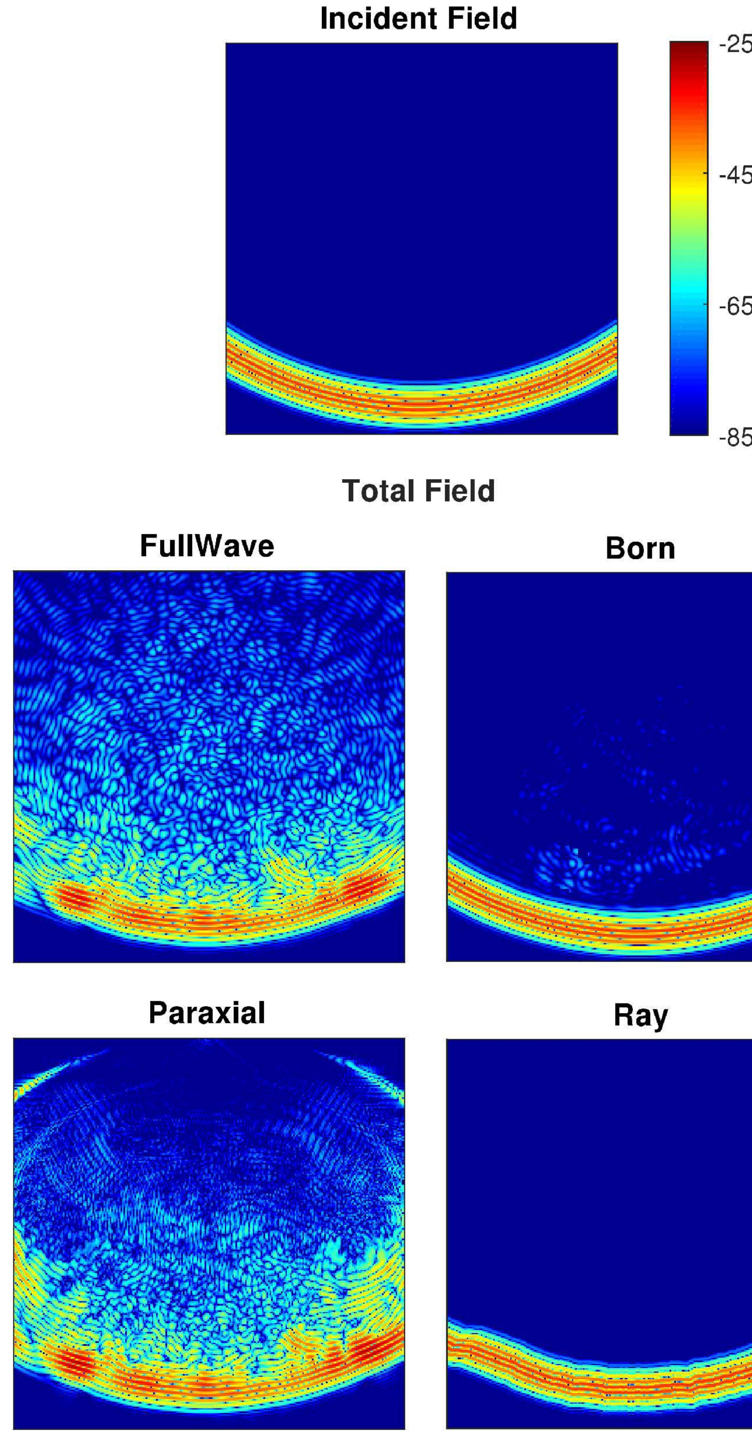
Integral equation:

$$\boxed{\text{Born approx.: } p \rightarrow p^{inc}}$$

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x}-\vec{x}')\, \chi(\vec{x}')\, p(\vec{x}')\, \mathrm{d}V(\vec{x}') \quad \text{with} \quad G(\vec{x}) = \frac{e^{-ik|\vec{x}|}}{4\pi|\vec{x}|}$$

Breast Ultrasound

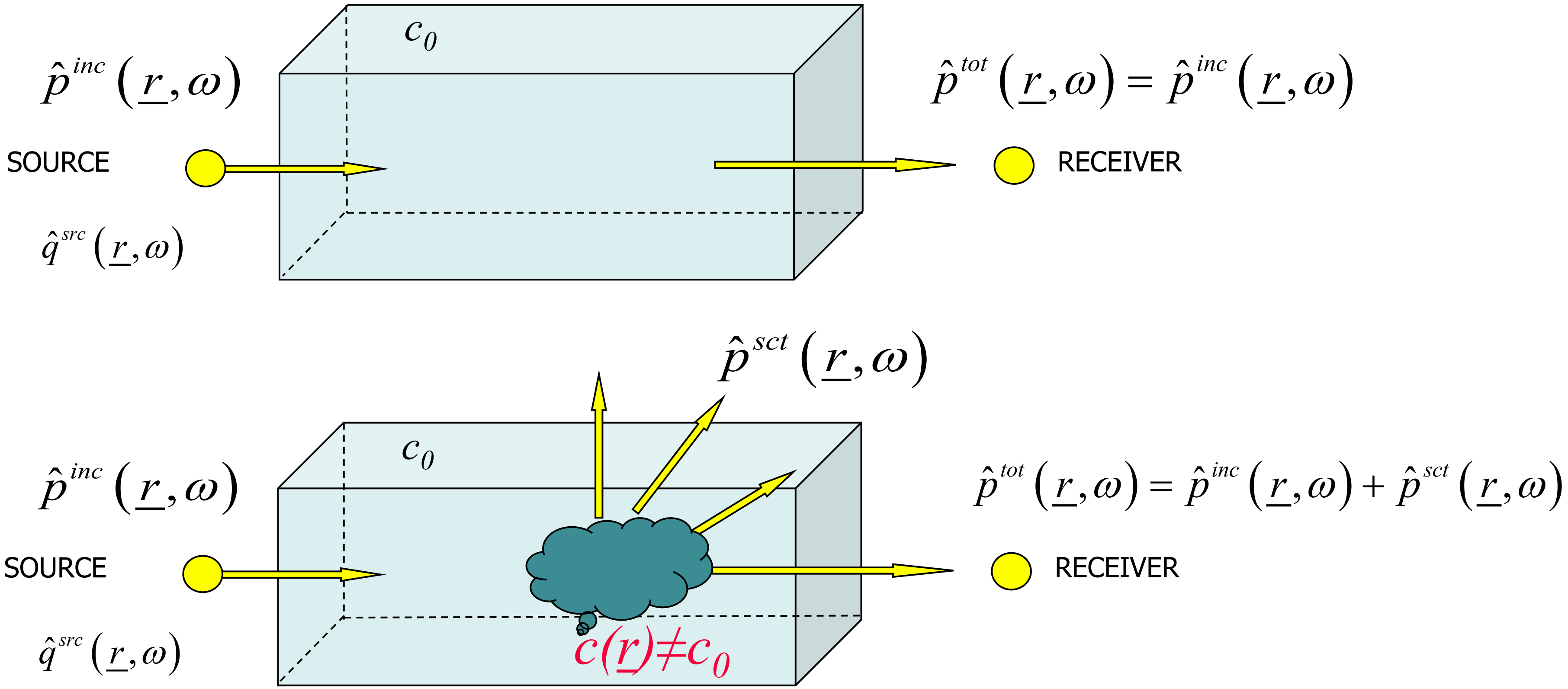




Ring with transducers

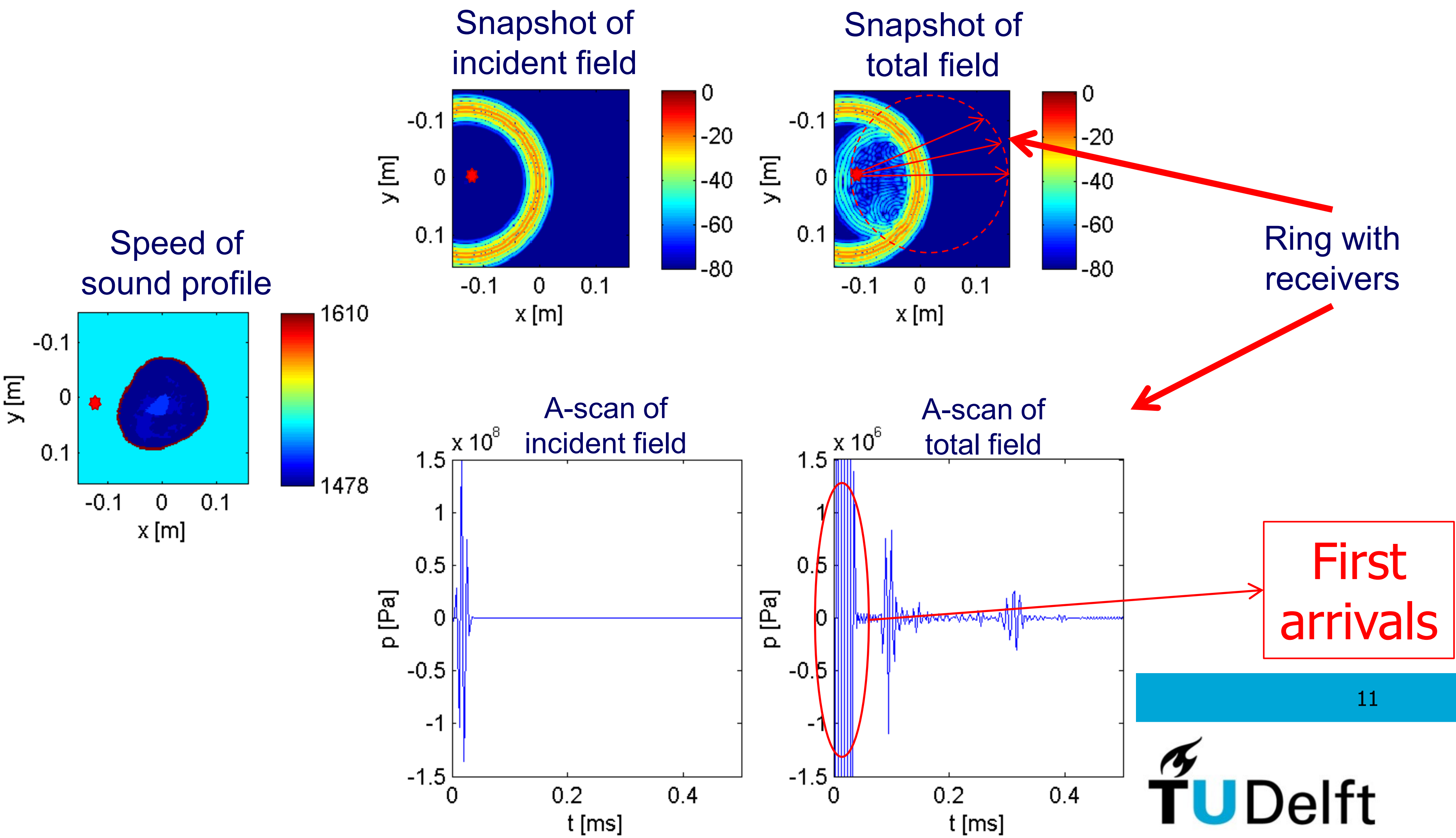
Forward versus Inverse Problem

- Incident, scattered and total field

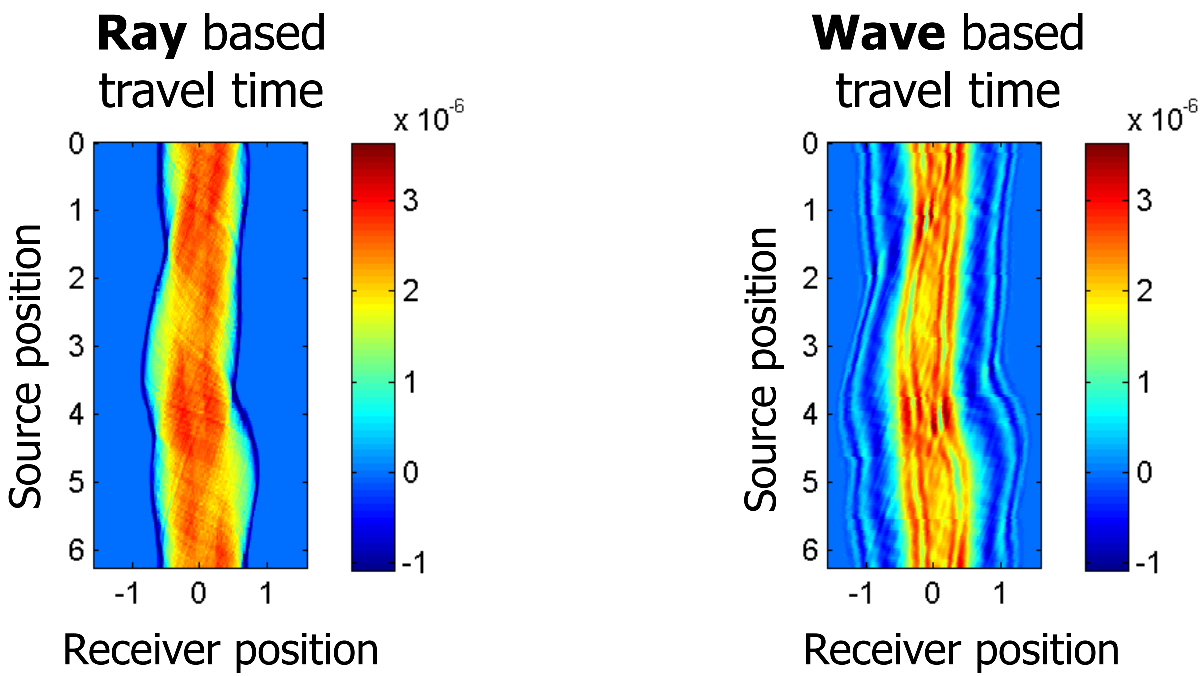


Breast ultrasound

Synthetic data obtained by solving forward problem for each source.



Inverse radon



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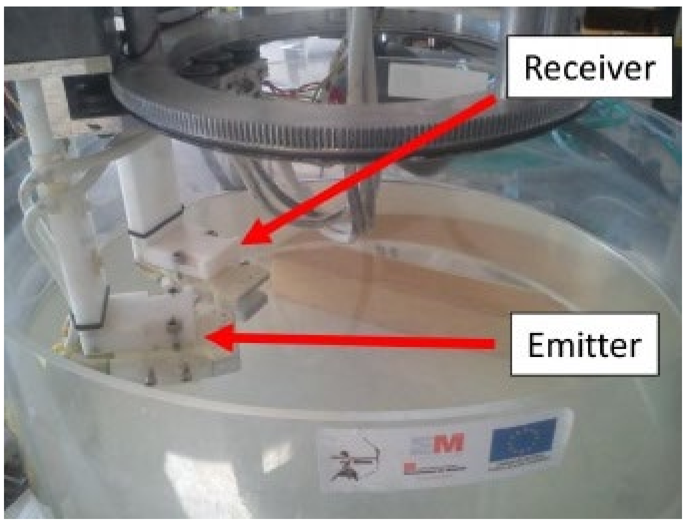
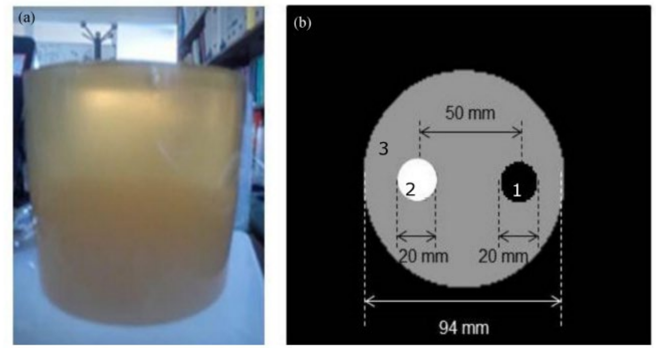
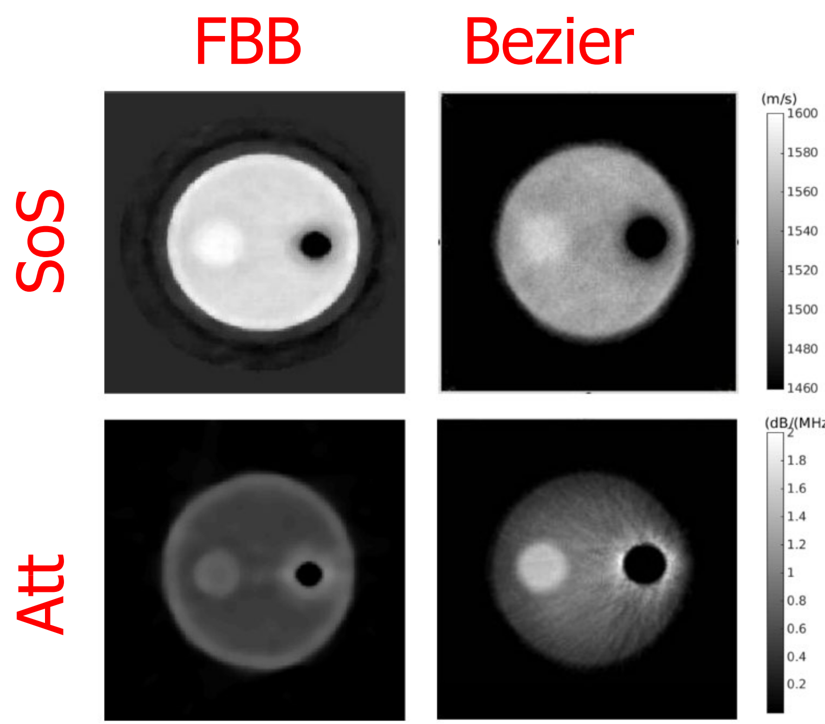
Inverse radon

- Bezier Curves: $B(t)=(1-t)^2P_0+2t(1-t)P_1+t^2P_2$
- MUBI system
  - >3MHz
  - $N_{Src/Rec}$ = N x 128



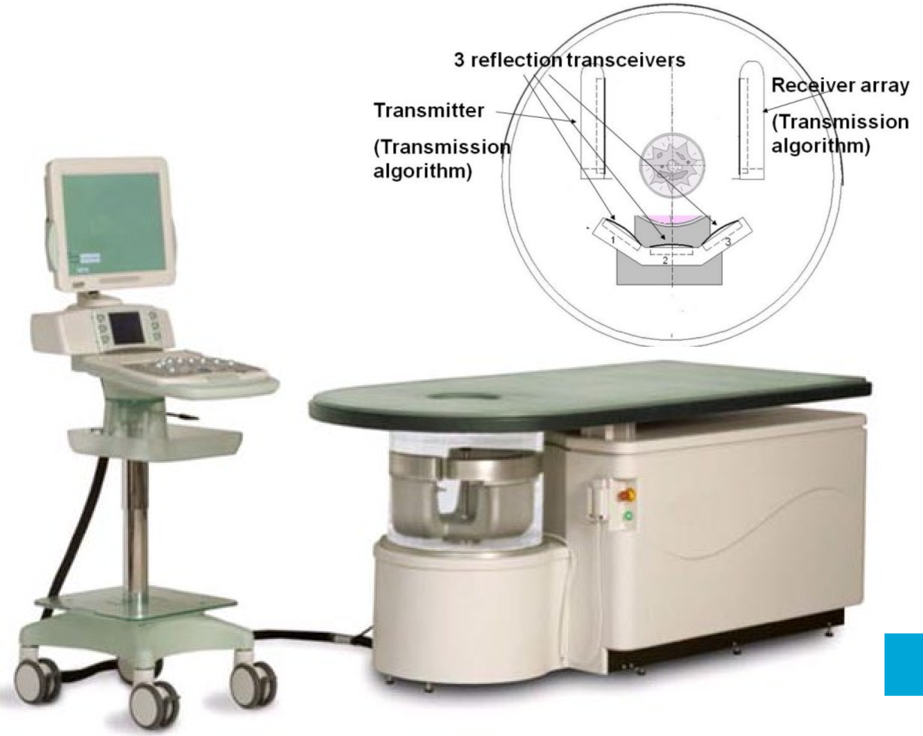


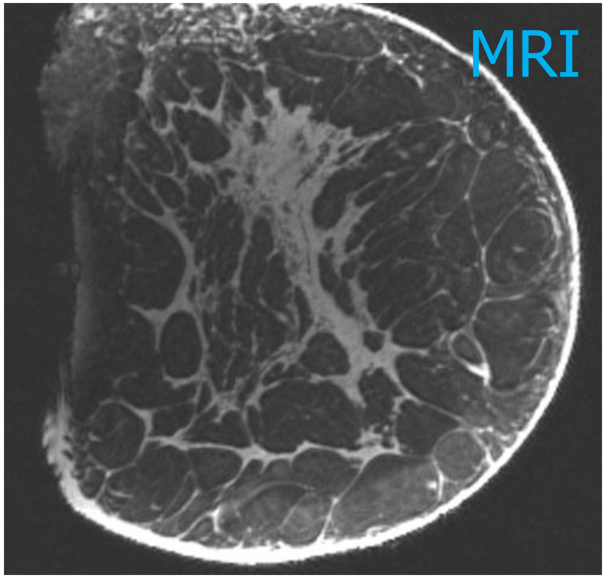


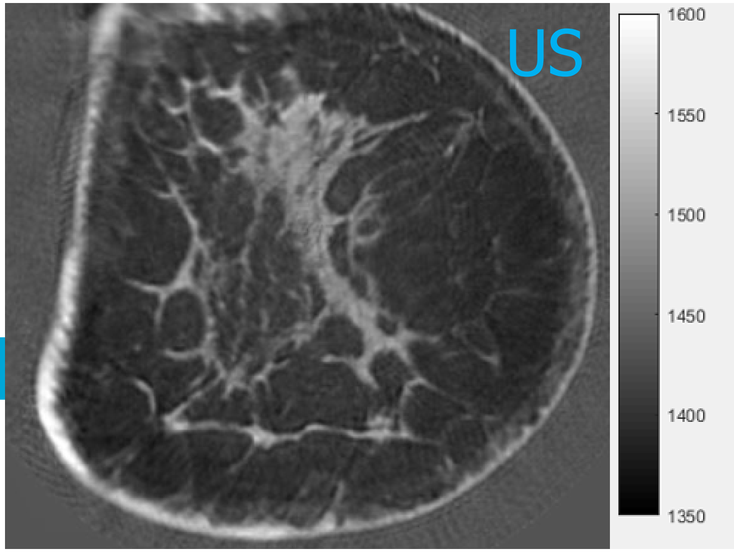
M. Perez-Liva, et al, "Real-Time Ultrasound Transmission Tomography based on Bézier Curves", Speyer 2017

Parabolic approximation (QT Ultrasound)

- $$p(k_x, k_y, z_0 + \Delta) = p(k_x, k_y, z_0)\, e^{-ik_z\Delta}$$
- Solution based on CG
- Out of plane scattering
- 3D formulation (with parts in 2D?)



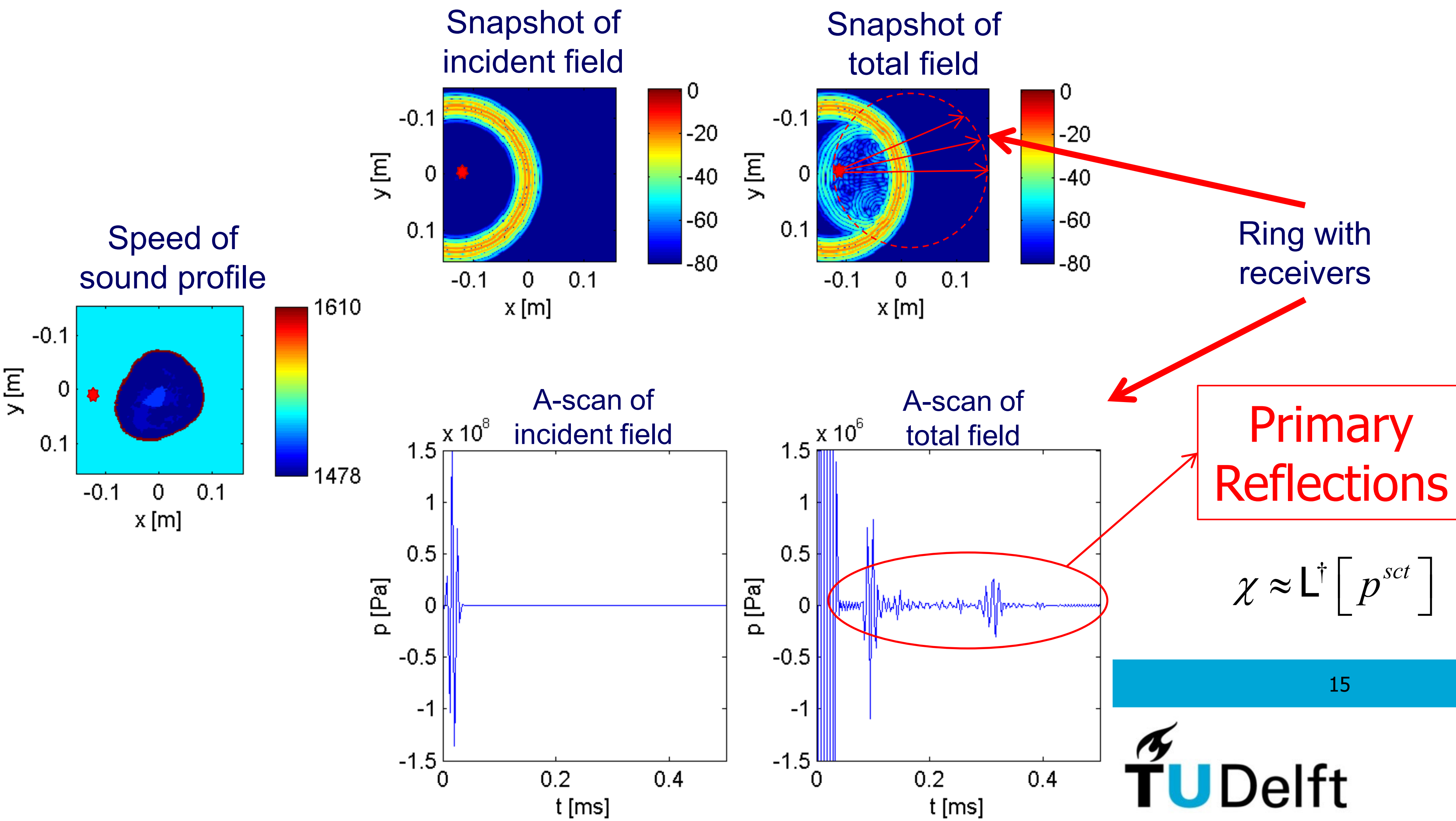




Wiskin et al. "3-D Nonlinear Acoustic Inverse Scattering: Algorithm and Quantitative Results", IEEE Trans. UFFC 2017.

Breast ultrasound

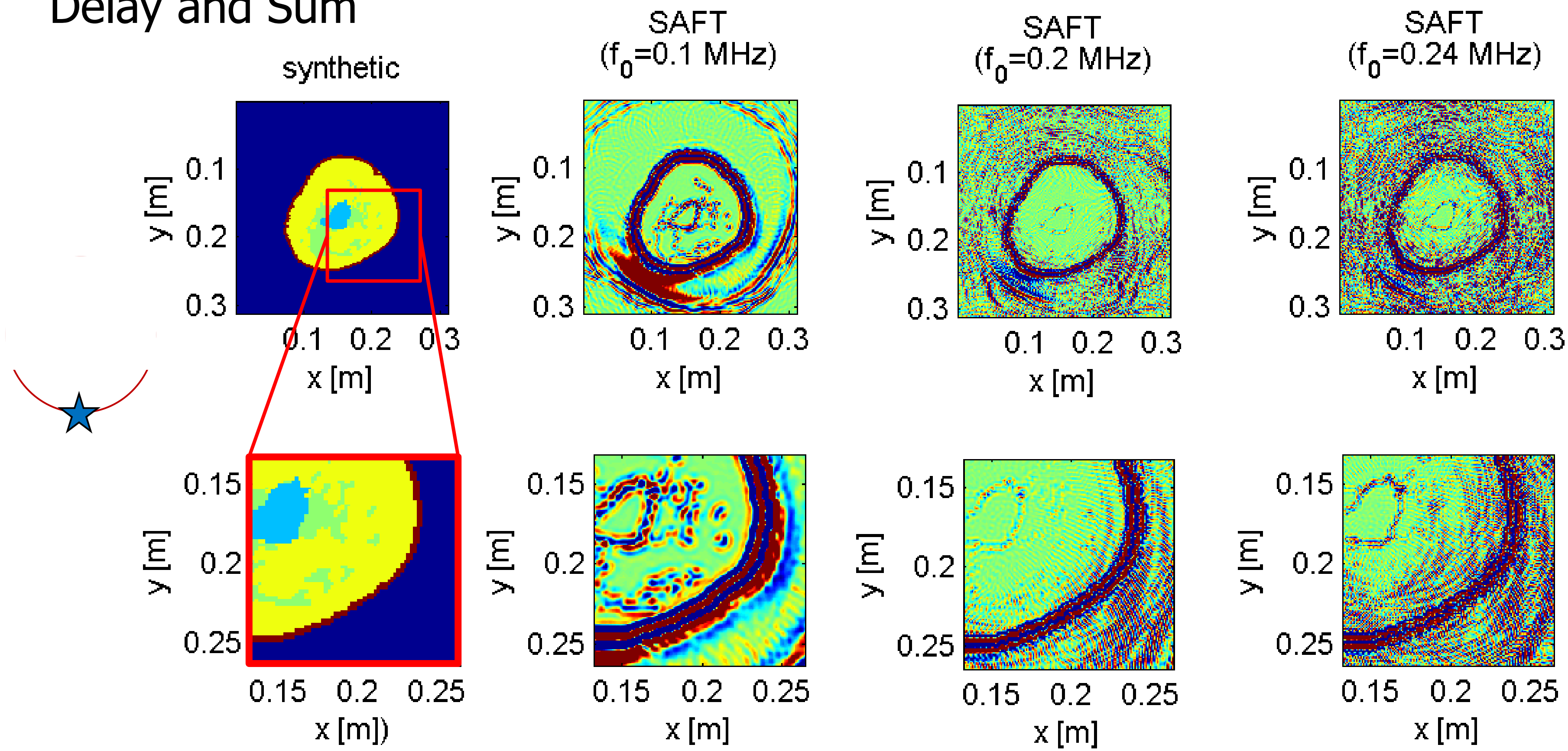
Synthetic data obtained by solving forward problem for each source.



$$\chi \approx \mathsf{L}^{\dagger}\left[p^{sct} \right]$$

Imaging: SAFT / SAR / DAS

Synthetic Aperture Focusing Technique
Synthetic Aperture Radar
Delay and Sum



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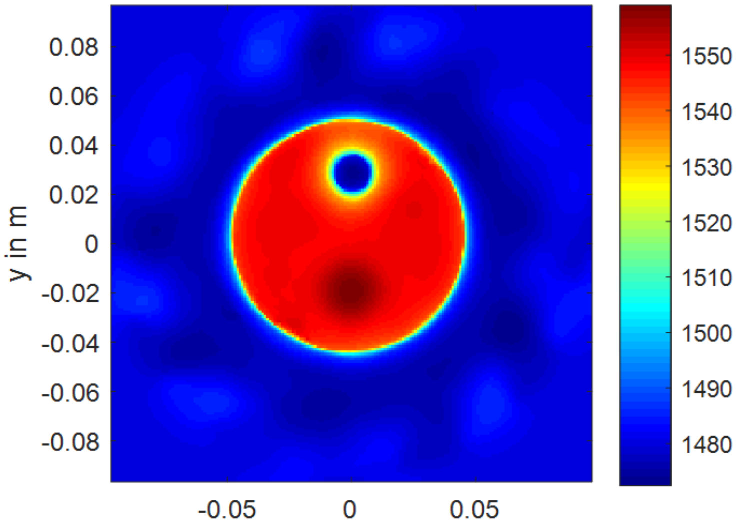
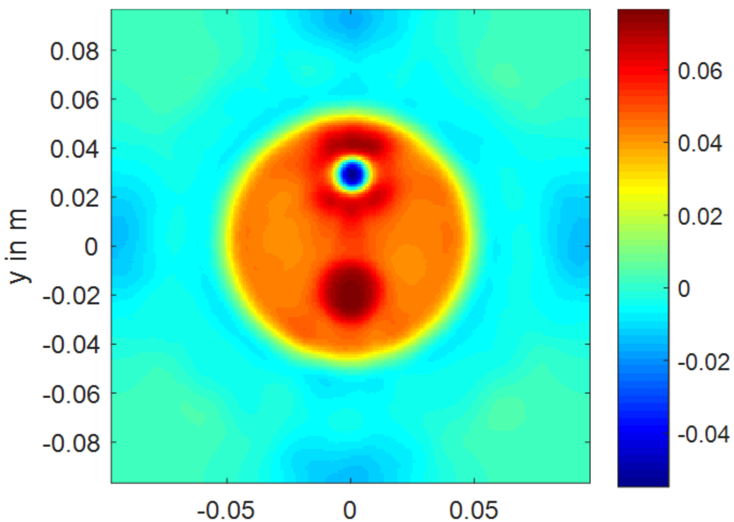
SAFT

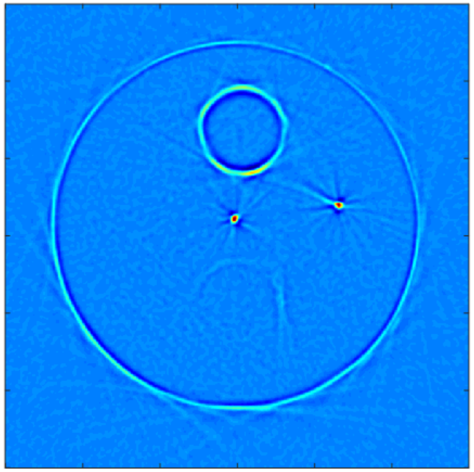
SoS

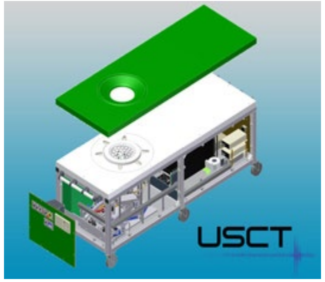


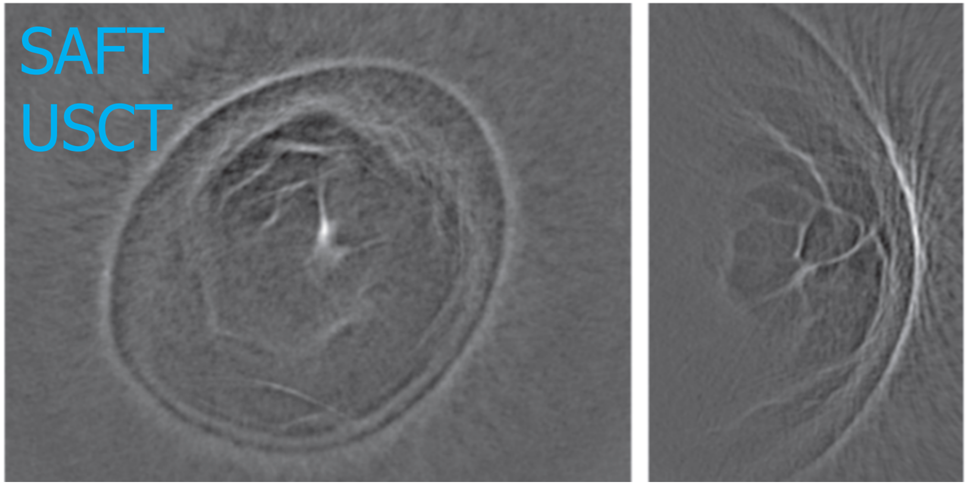
Attenuation



Saft (MUBI-system)







N.V. Ruiter, et. al. "The USCT reference database", Speyer (2017)
N.V. Ruiter, et. al. "Fast reflectivity imaging in 3D using SAFT", Speyer (2017)

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Acoustic wave equation based inversion

Wave equation:

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})}\partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

Helmholtz equation:

$$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c_{bg}^2} p(\vec{x}) = -S^{pr}(\vec{x}) + \omega^2 \underbrace{\left(\frac{1}{c_{bg}^2} - \frac{1}{c^2(\vec{x})}\right)}_{=\chi(\vec{x})} p(\vec{x})$$

Inverse problem – Born inversion (linearized inversion)

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G\,(\vec{x} - \vec{x}')\, \boxed{\chi(\vec{x}')\; p(\vec{x}')}\, \mathrm{d}V(\vec{x}')$$

Born Approximation:

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G\,(\vec{x} - \vec{x}')\; \chi(\vec{x}')\; p^{inc}(\vec{x}')\; \mathrm{d}V(\vec{x}')$$

$$p - p^{inc} = -\omega^2 G * \;[\chi p^{inc}]$$

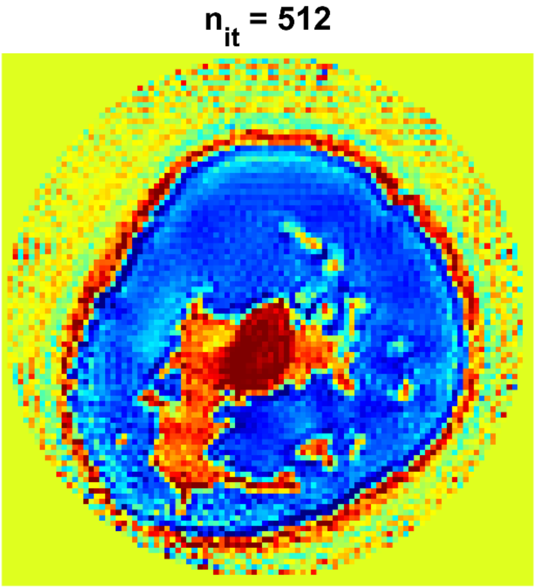
(b=Ax) Conjuge gradient scheme

$$r_n = p^{sct} - \omega^2 G * \;[\chi_n p^{inc}]$$

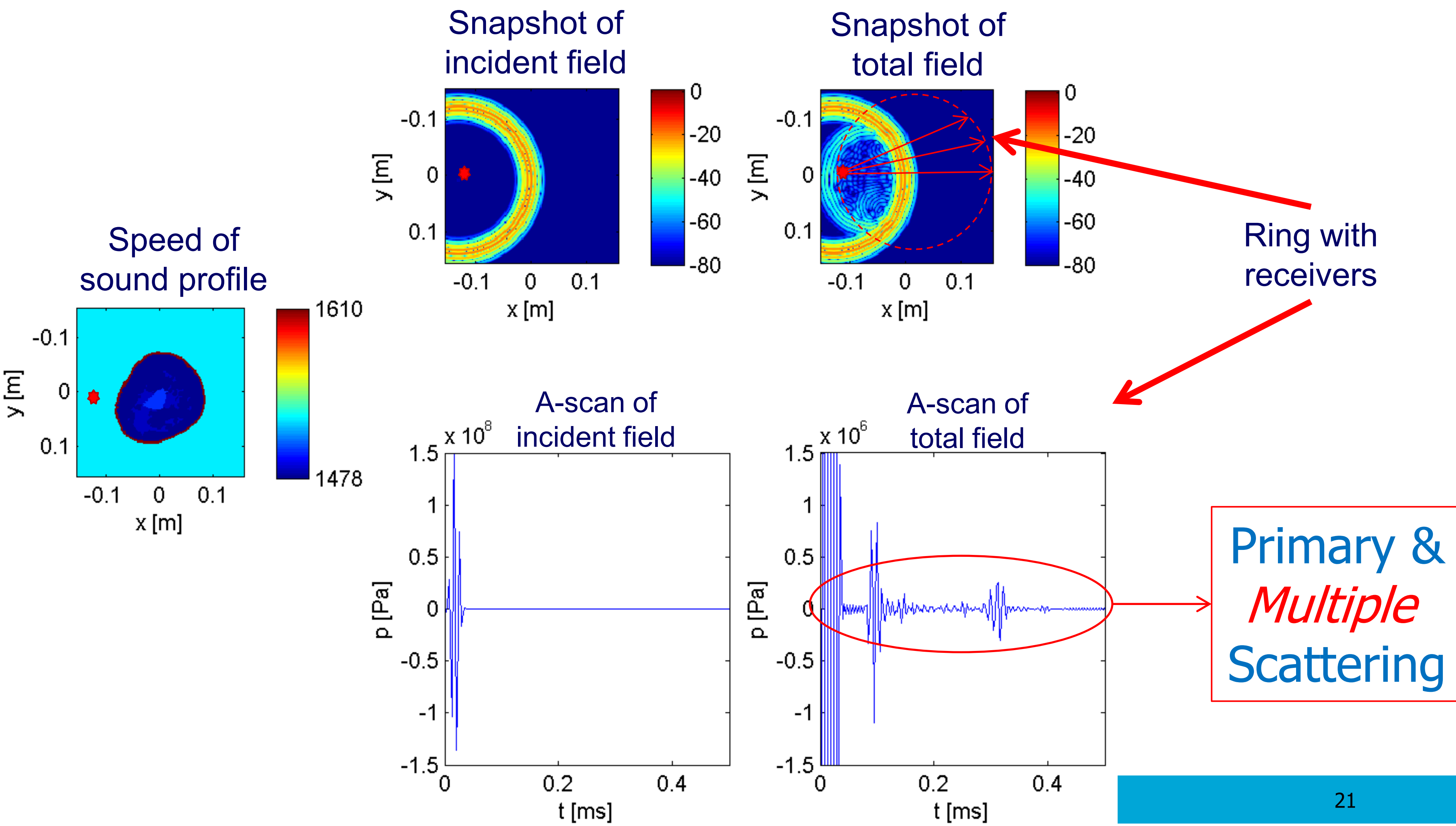
$$\chi_n = \chi_{n-1} + d_n$$

Linear or Born Inversion

Linear Inversion is an unstable process due to the Born-approximation.



Breast ultrasound



Wave equation

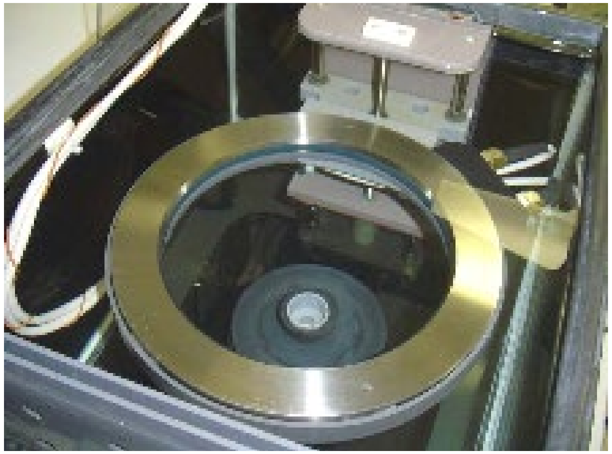
$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})}\partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

- 2-D inversion

  (3D too expensive)

- CURE system

  (Karmanos Cancer Institute)



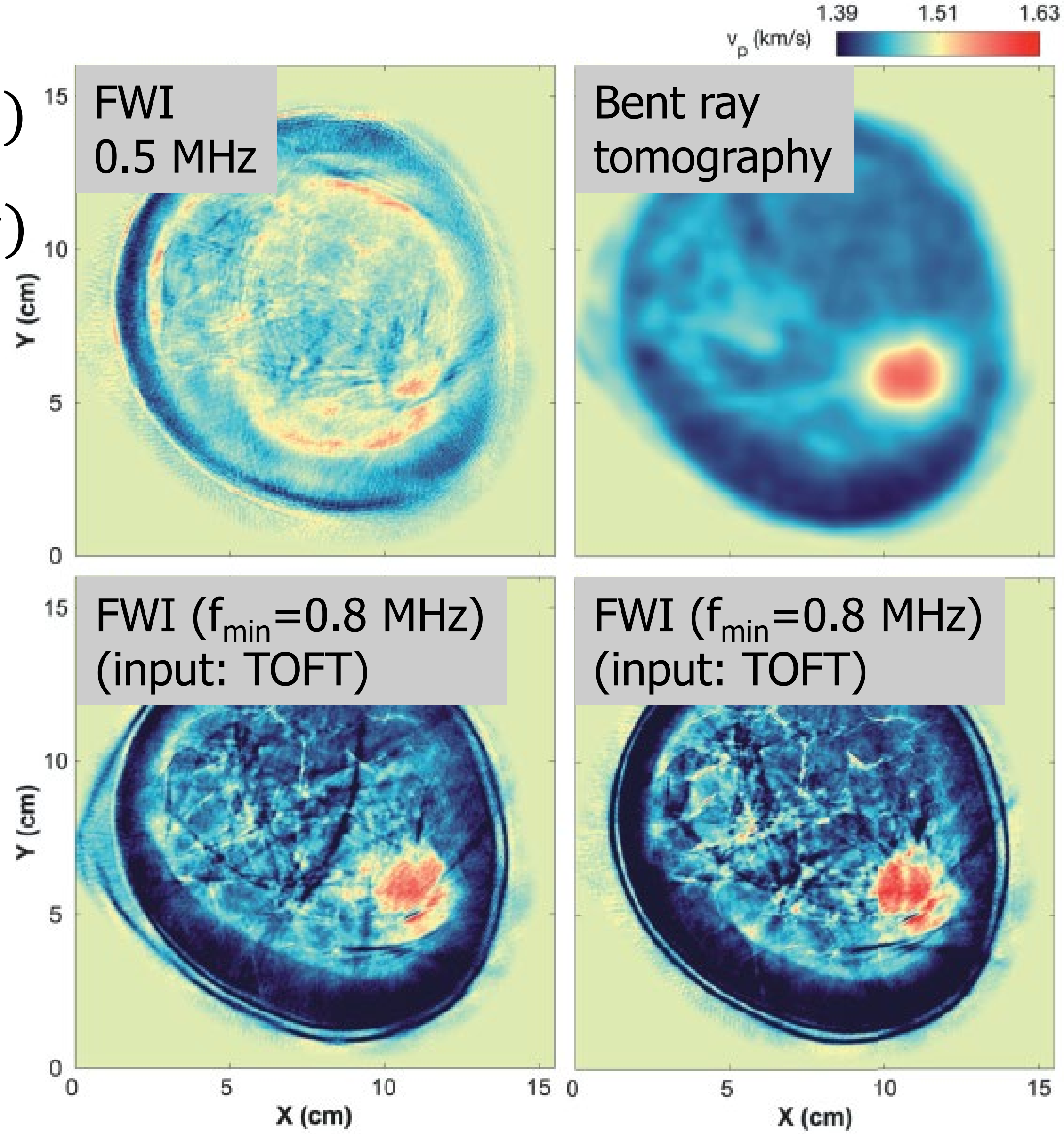


Figure 7: Recovered models of speed of sound obtained from the 2D *in vivo* data after using a) FWI at $f_{min} = 500kHz$ and a constant water starting model, b) bent-ray tomography and FWI with the time-of-flight model as a starting model at c) $f_{min} = 800kHz$ and d) at $f_{min} = 500kHz$. The maximum frequency for all FWI tests is $1.75MHz$.

Agudo, et al, "3D imaging of the breast using FWI", Speyer (2017)

Inverse problem – non-linear inversion
(full-waveform inversion – FWI – contrast source inversion – CSI)

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G\,(\vec{x} - \vec{x}')\ \chi(\vec{x}')\ p(\vec{x}')\ \mathrm{d}V(\vec{x}')$$

Born Approximation:

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G\,(\vec{x} - \vec{x}')\ \chi(\vec{x}')\ p^{inc}(\vec{x}')\ \mathrm{d}V(\vec{x}')$$

Full non–linear inversion:

$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G\,(\vec{x} - \vec{x}')\ w(\vec{x}')\ \mathrm{d}V(\vec{x}') \qquad p^{tot} = p^{inc} - \omega^2 G * w$$

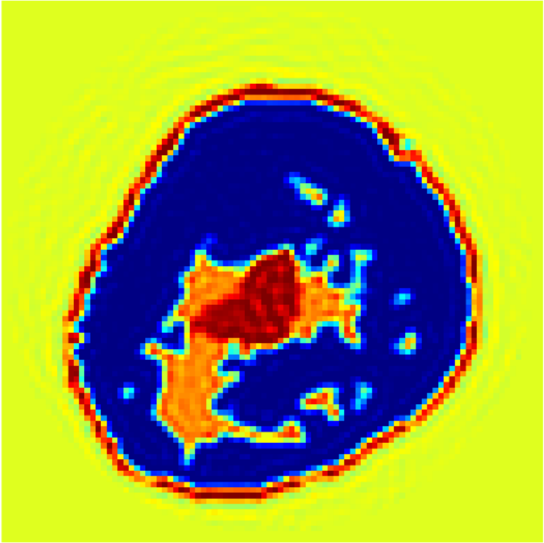
$$w(\vec{x}') = \chi(\vec{x}')p(\vec{x}') \qquad w = \chi p^{tot}$$

$$Err_n = \frac{\left\|p - \left(p^{inc} - \omega^2 G * w_n\right)\right\|^2}{\left\|p^{inc}\right\|^2} + \frac{\left\|w_n - \chi_{n-1}\left(p^{inc} - \omega^2 G * w_n\right)\right\|^2}{\left\|\chi_{n-1} p^{inc}\right\|^2}$$

Non-Linear Inversion

- Non-linear Inversion is a stable process as it uses the full wave equation.

$n_{it} = 1024$



Non-linear inversion

$$p^{tot}(\vec{r}) = p^{inc}(\vec{r}) - \int G\,(\vec{r} - \vec{r}')\ \omega^2 \chi(\vec{r}')\ p^{tot}(\vec{r}')\ \mathrm{d}V$$

Born Approximation:

$$\hat{p}^{tot}(\vec{r}) = \hat{p}^{inc}(\vec{r}) - \omega^2 \int \hat{G}\,(\vec{r} - \vec{r}')\ \chi(\vec{r}')\ \hat{p}^{inc}(\vec{r}')\ \mathrm{d}V$$

Full non−linear inversion:

$$\hat{p}^{tot}(\vec{r}) = \hat{p}^{inc}(\vec{r}) - \omega^2 \int \hat{G}\,(\vec{r} - \vec{r}')\ \hat{w}(\vec{r}')\ \mathrm{d}V \qquad p^{tot} = p^{inc} - G * w$$

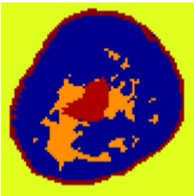
$$\hat{w}(\vec{r}') = \chi(\vec{r}')\hat{p}^{tot}(\vec{r}') \qquad w = \chi p^{tot}$$

$$Err^{(n)} = \left[\frac{\left\| p^{tot} - p^{inc} + G * w^{(n)} \right\|}{\left\| p^{inc} \right\|} + \frac{\left\| w^{(n)} - \chi^{(n-1)}\left(p^{inc} - G * w^{(n)}\right) \right\|}{\left\| \chi^{(n-1)} p^{inc} \right\|} \right] + (\ldots)$$

Regularization

- Total Variation
- sparsity
  - w
  - χ

5% noise

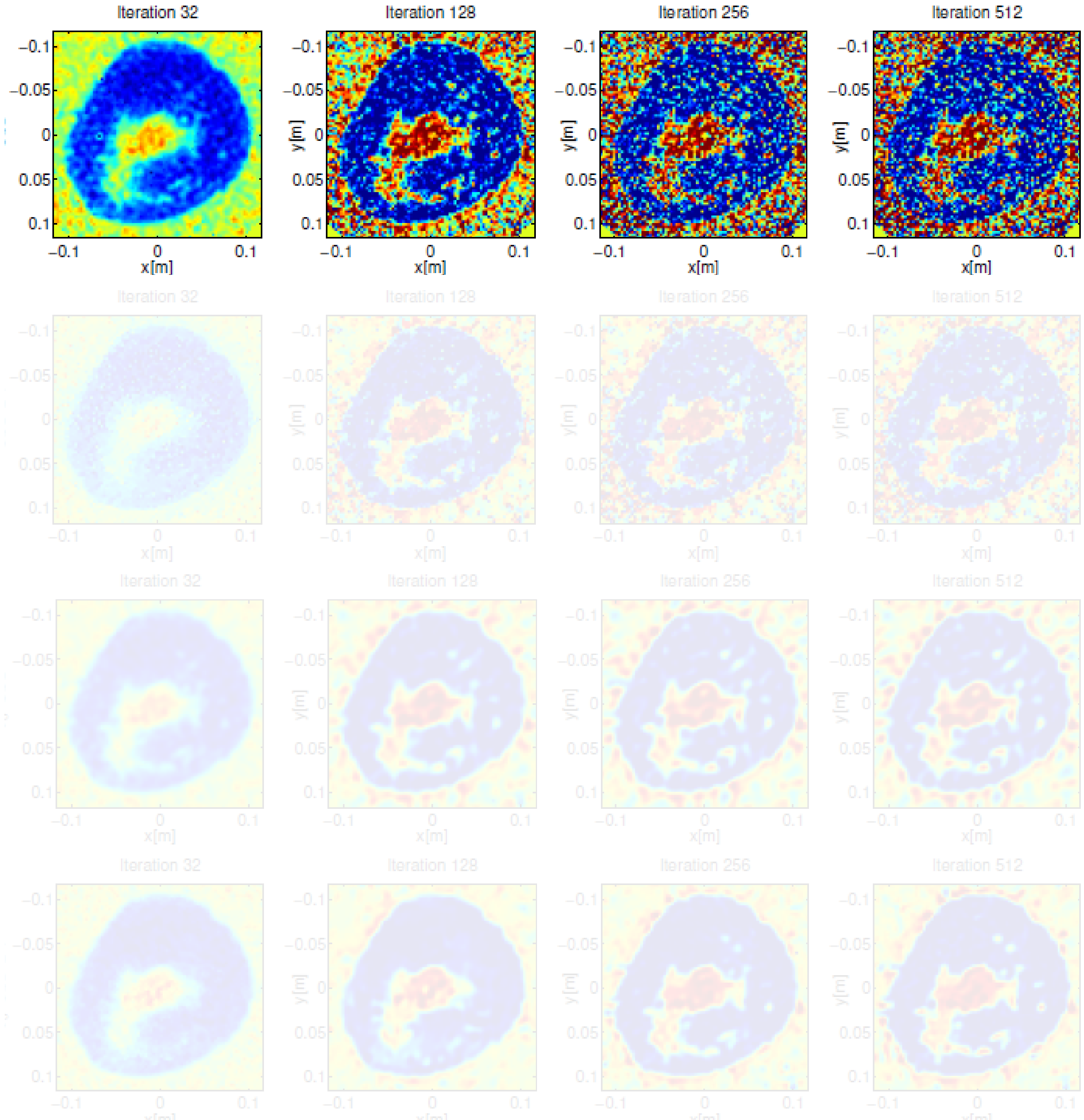


$$E_{TV} = \eta_{TV} \int_{\vec{x} \in D} \sqrt{\left|\nabla \chi_n(\vec{x})\right|^2}\, dA$$

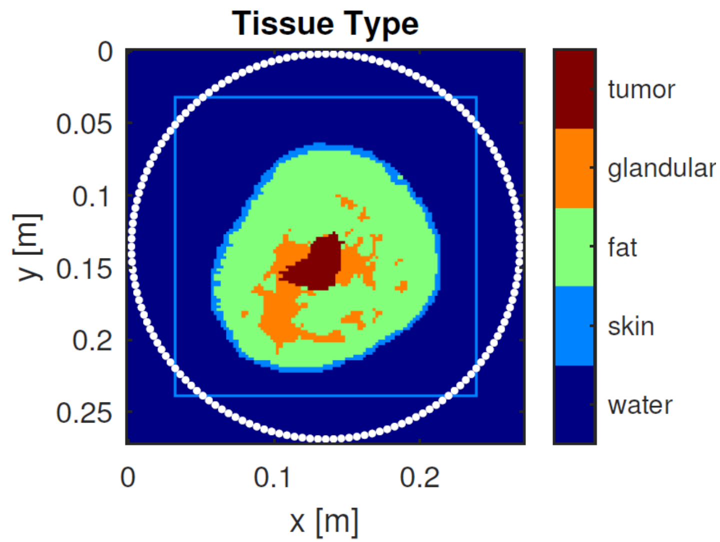
$$\eta_S E_S + \eta_D E_D$$

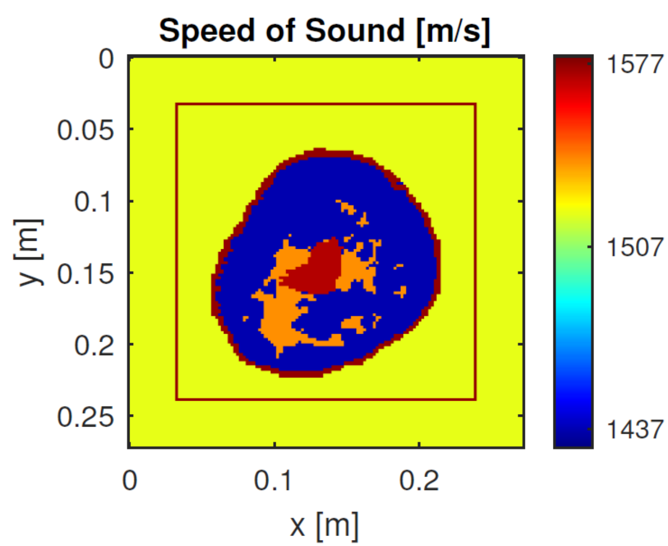
$$s.t. \begin{cases} \left\| F\left[\hat{w}_j\right] \right\|_0 < s_1 \\ \left\| \Phi\left[\chi\right] \right\|_0 < s_2 \end{cases}$$

All tools

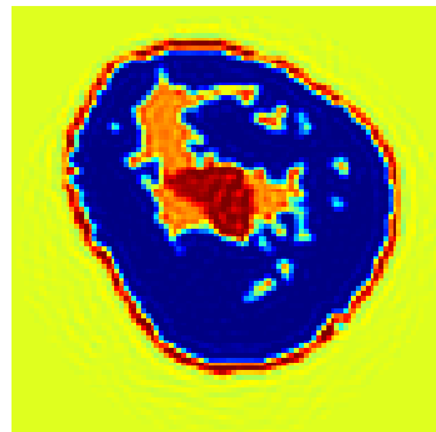


Frequency versus time domain

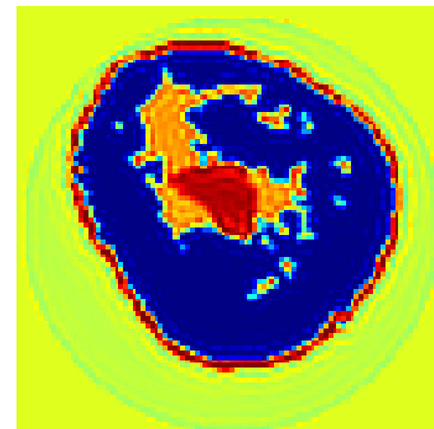




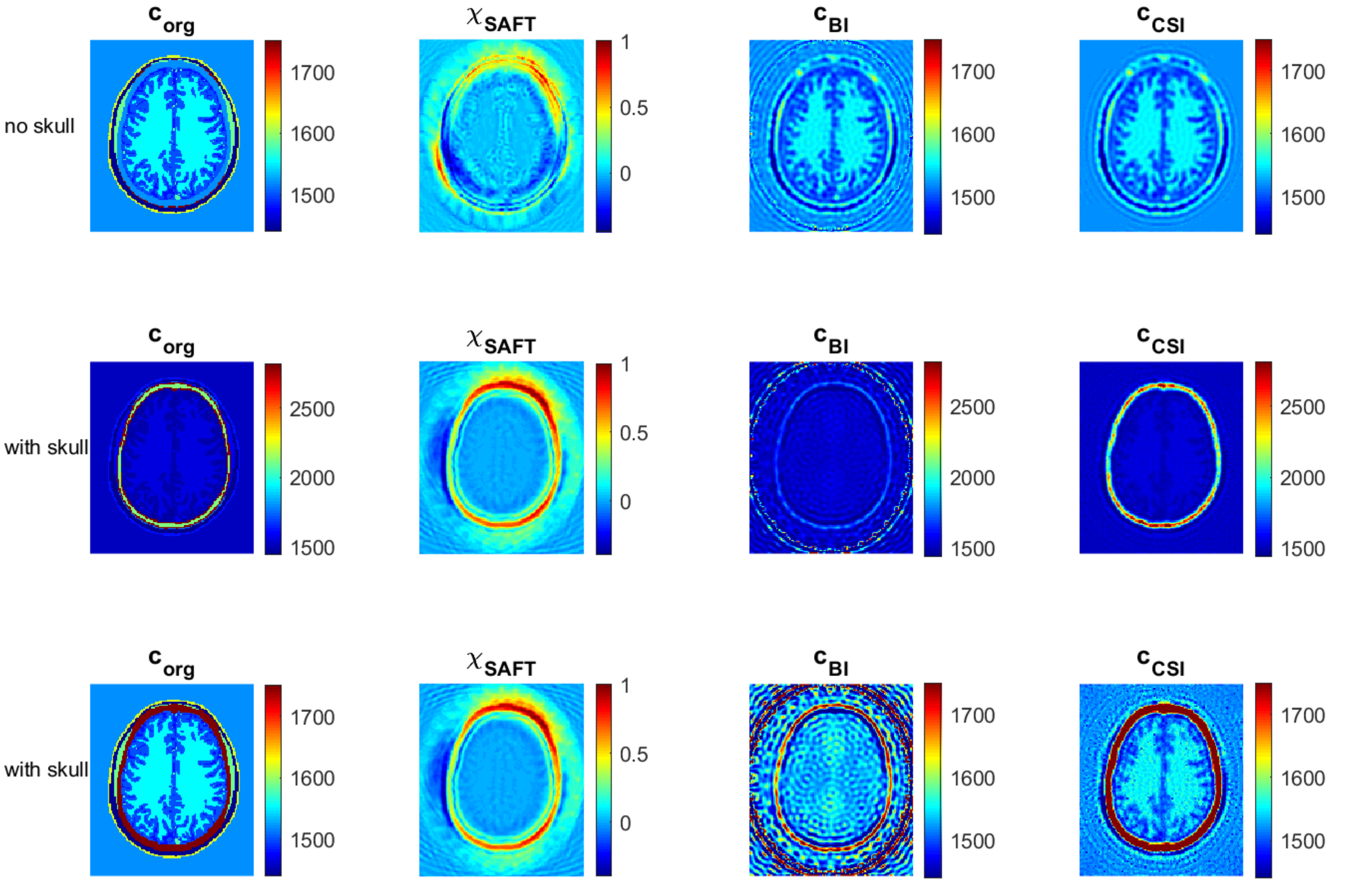
Frequency domain



Time domain

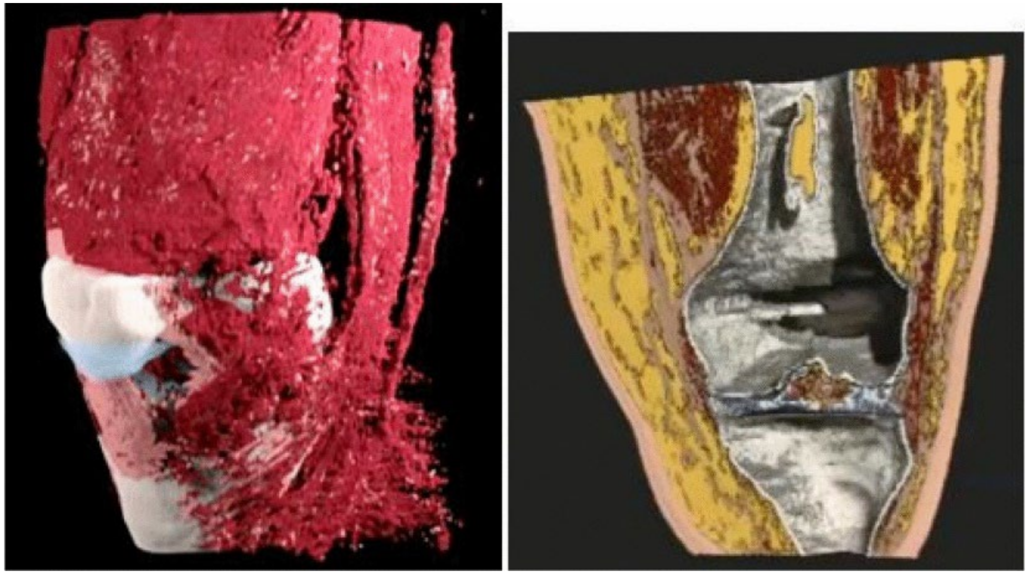


Transcranial Ultrasound



Imaging other body parts

- State of the art: 3-D FWI of breasts and other body parts as well the earth.



3D rendering of the SOS reconstruction of the human knee.

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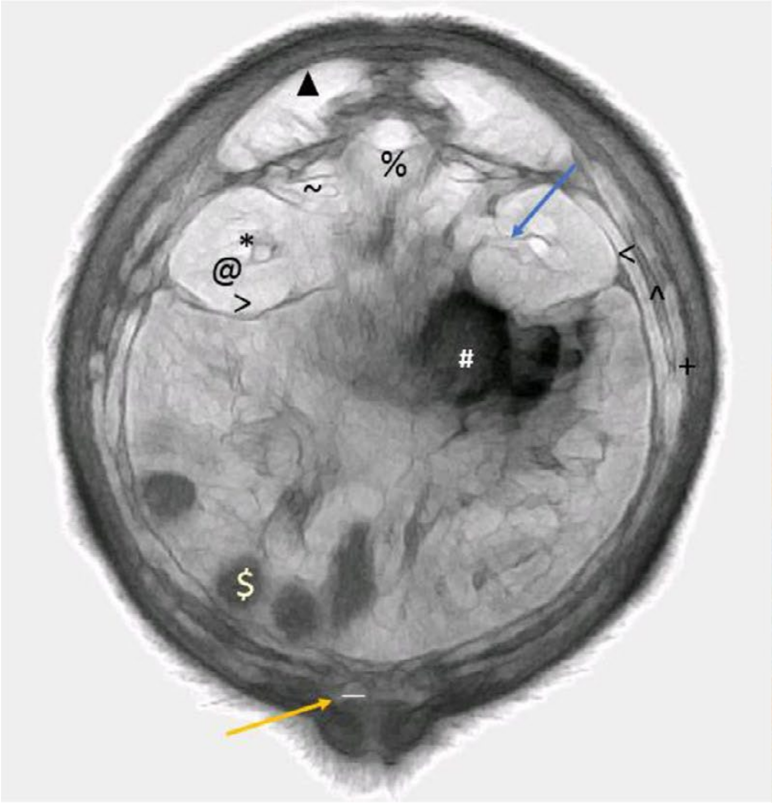
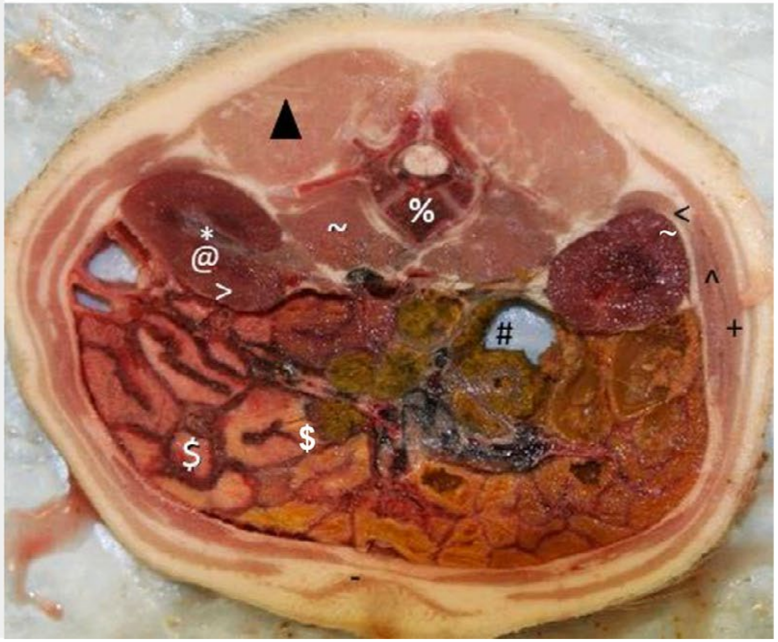
Wiskin, *et al.* "Full wave 3D inverse scattering transmission ultrasound tomography in the presence of high contrast," *Sci Rep* **10**, 20166 (2020)

Whole body imaging

- Anatomical correlation of pig abdomen

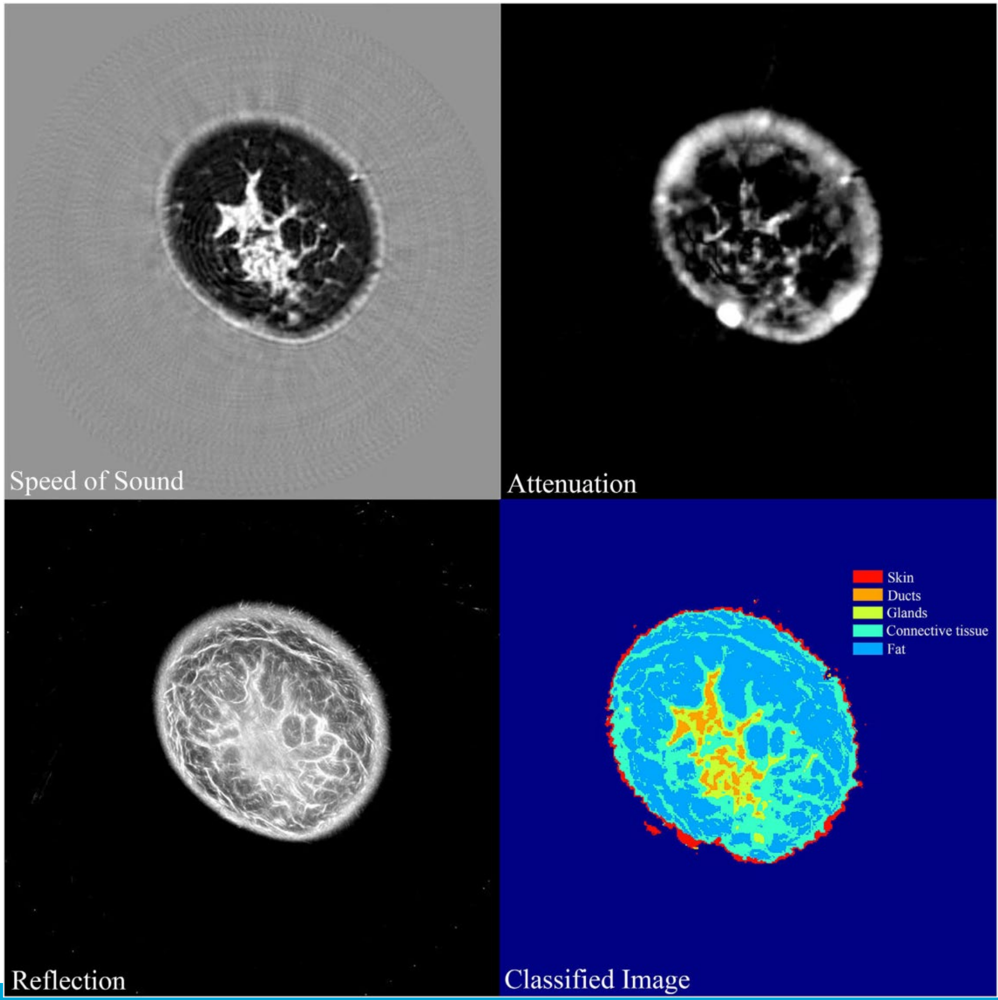




Wiskin et al., "Whole-Body Imaging Using Low Frequency Transmission Ultrasound, Academic Radiology (2023)

Machine learning for tissue classification

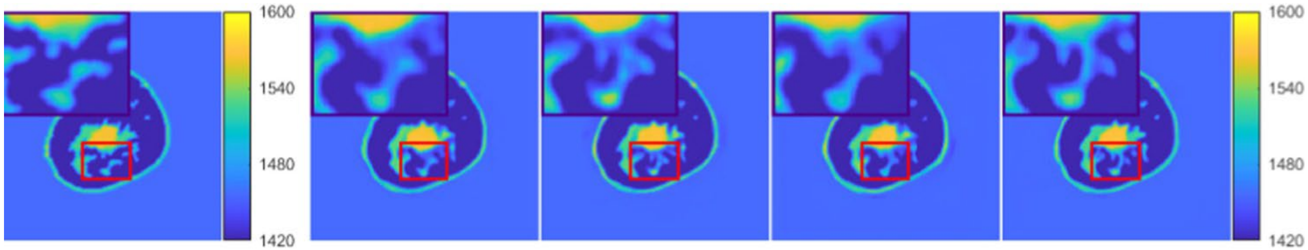
- QT Ultrasound
  - Based on 13 breasts
  - Tissue parameters only (?)
- Problems will occur to apply method to different systems
- Delphinus:
  Tissue parameters and Texture.



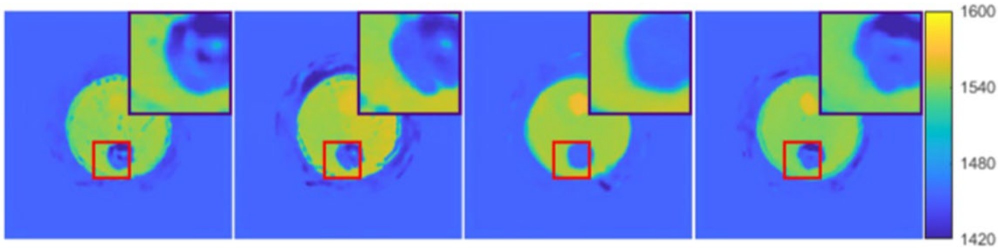
B. Malik, et al. "Objective breast tissue image classification using Quantitative Transmission ultrasound tomography," Scientific Reports, 2016

Machine learning for inversion

On synthetic data



On real data



Zhao *et al* 2023, "Simulation-to-real generalization for deep-learning-based refraction-corrected ultrasound tomography image reconstruction," *Phys. Med. Biol.* **68** (2023)

Redatuming

- Computational load FWI is major downside.
- In seismics, Rayleigh II is used to propagate the wave field downwards. Not feasible for curved surfaces.
- Kirchhoff would work on curved surface … but requires knowledge of both the pressure field and the velocity field!
- => Solution: Describe measurements using Hankel functions.

Redatuming

$$\left(\nabla^2 - \frac{\omega^2}{c^2}\right)\hat{p}(x,y,\omega) = \left(r^2\partial_r^2 + r\partial_r + \partial_\phi^2 + r^2\frac{\omega^2}{c^2}\right)\hat{p}(r,\phi,\omega) = 0$$

$$\hat{p}(r,\phi,\omega) = \Gamma(r,\omega)\Phi(\phi)$$

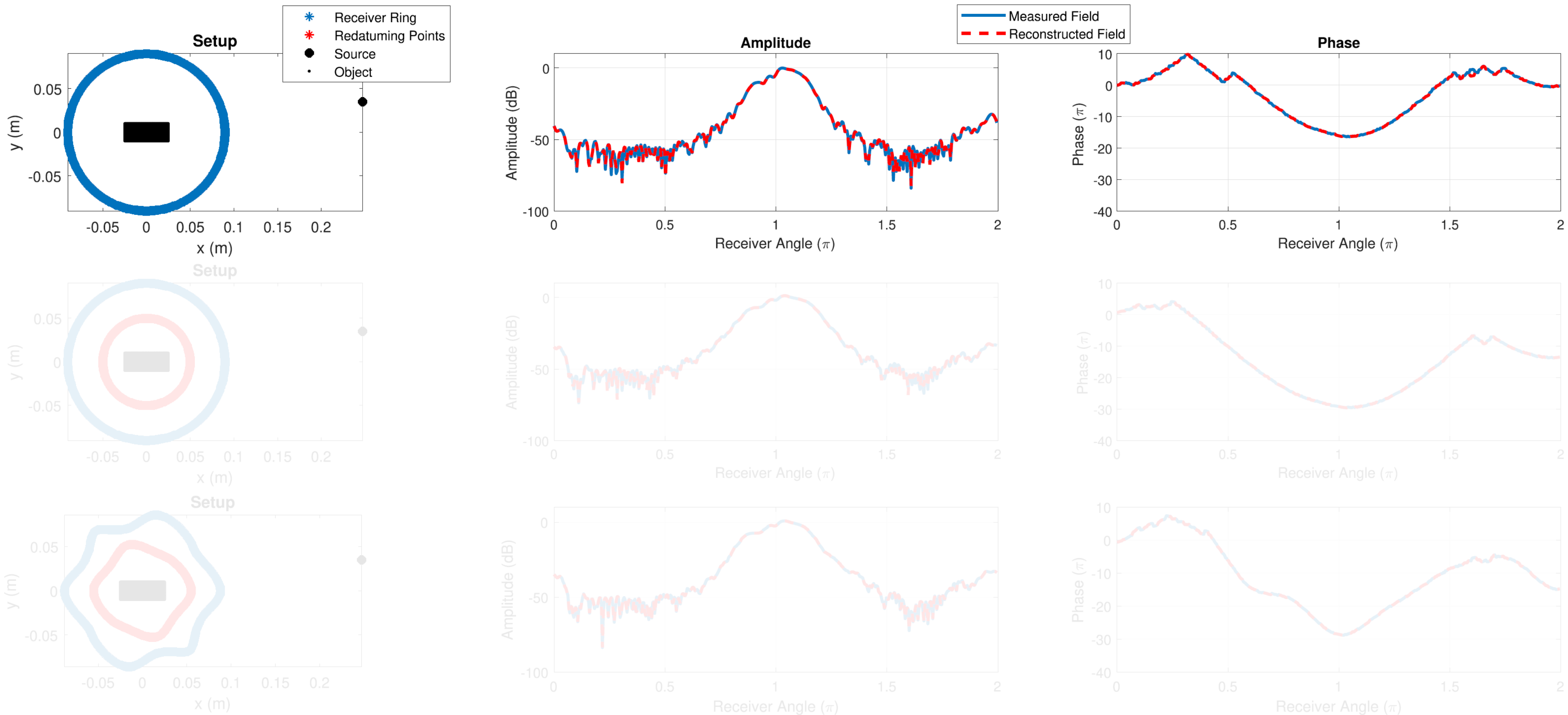
$$\Phi_n(\phi) = a_n e^{in\phi} \quad \text{with} \quad n \in \mathbb{Z}$$

$$\left(r^2\partial_r^2 + r\partial_r + \left(r^2\frac{\omega^2}{c^2} - n^2\right)\right)\Gamma_n(r,\omega) = 0$$

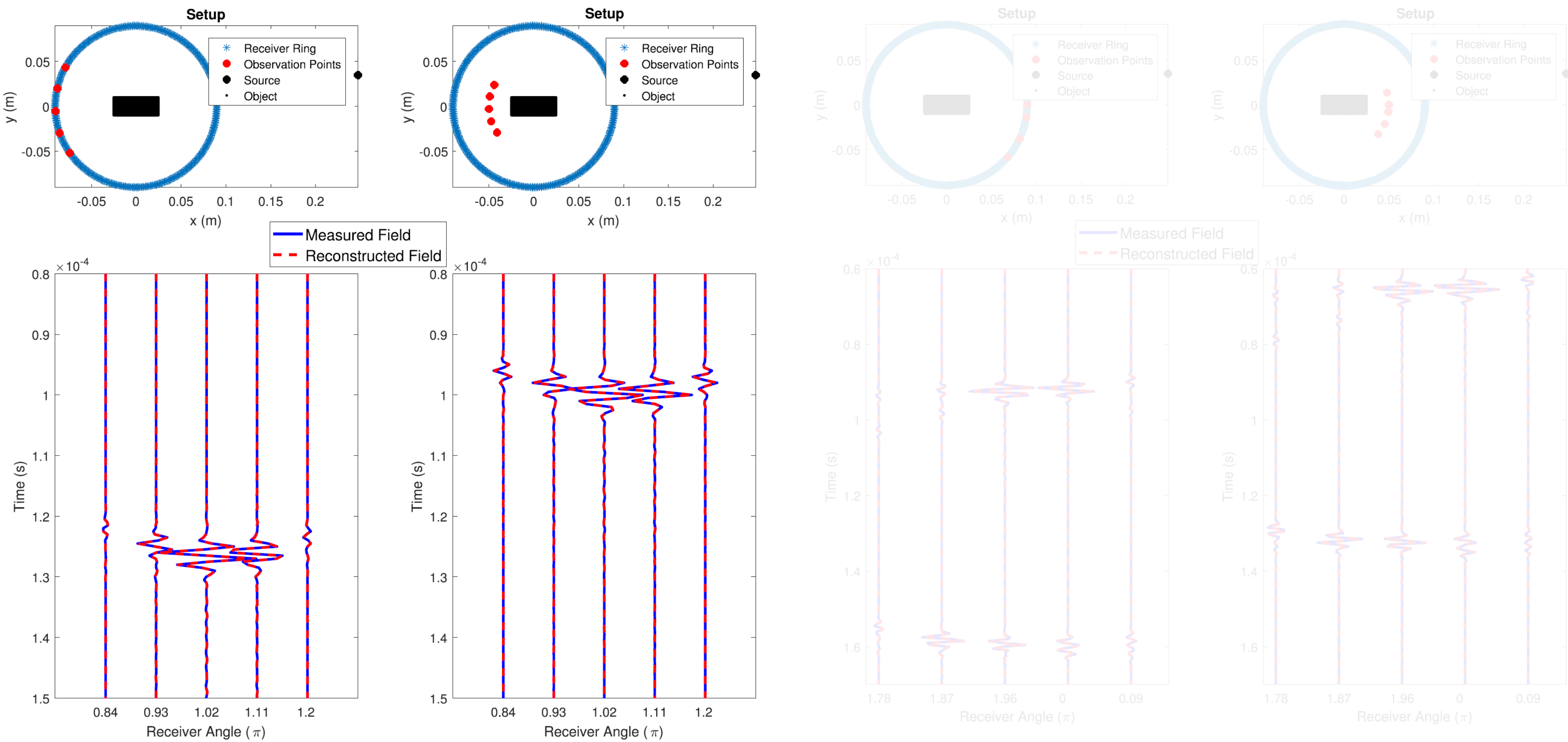
$$\Gamma_n(r,\omega) = b_{n,1}(\omega)H_n^{(1)}\left(\frac{\omega r}{c}\right) + b_{n,2}(\omega)H_n^{(1)}\left(\frac{\omega r}{c}\right)$$

$$\boxed{p_n(r,\phi) = a_n H_n^{(1)}\left(\frac{\omega r}{c}\right)e^{in\phi}}$$

Redatuming

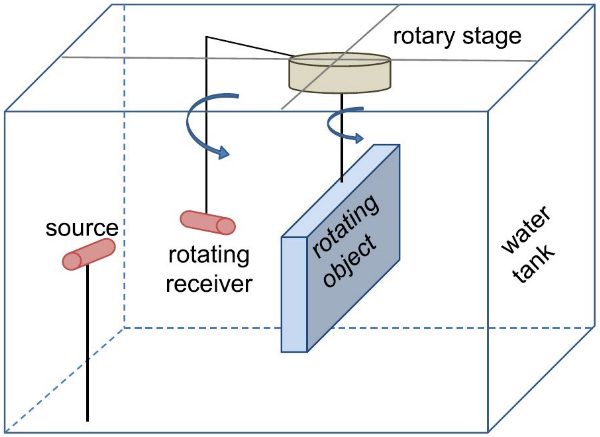

Setup
Receiver Ring
Redatuming Points
Source
Object
y (m)
x (m)
0.05
0
-0.05
-0.05
0
0.05
0.1
0.15
0.2
Measured Field
Reconstructed Field
Amplitude
Amplitude (dB)
0
-50
-100
Receiver Angle (π)
0
0.5
1
1.5
2
Phase
Phase (π)
10
0
-10
-20
-30
-40
Receiver Angle (π)

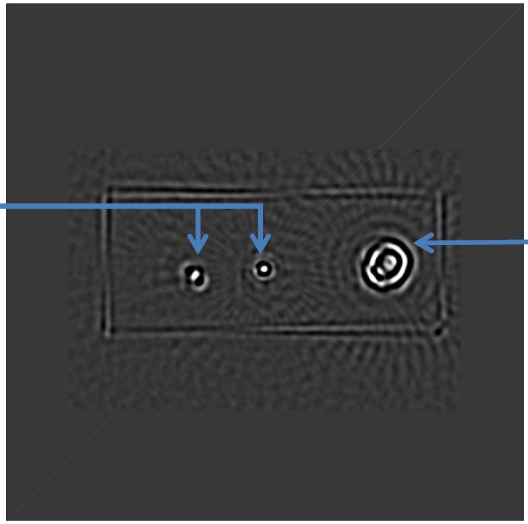
Redatuming

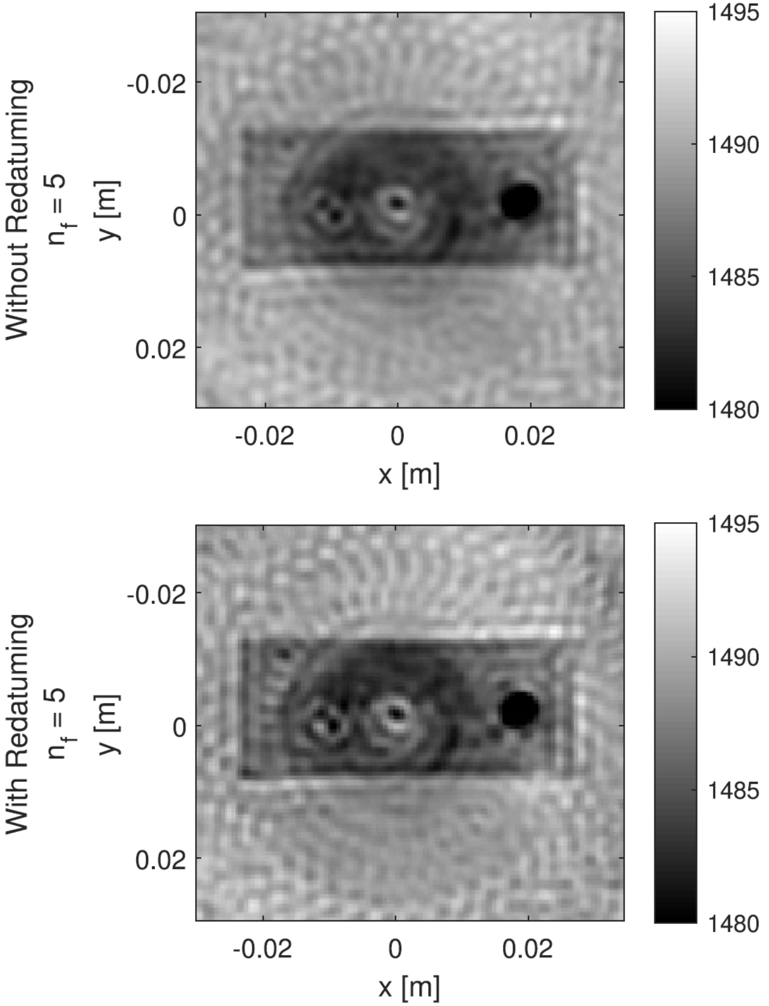

Setup
Receiver Ring
Observation Points
Source
Object
y (m)
x (m)
Measured Field
Reconstructed Field
Time (s)
Receiver Angle (π)
$\times 10^{-4}$

Redatuming

Contrast Source inversion

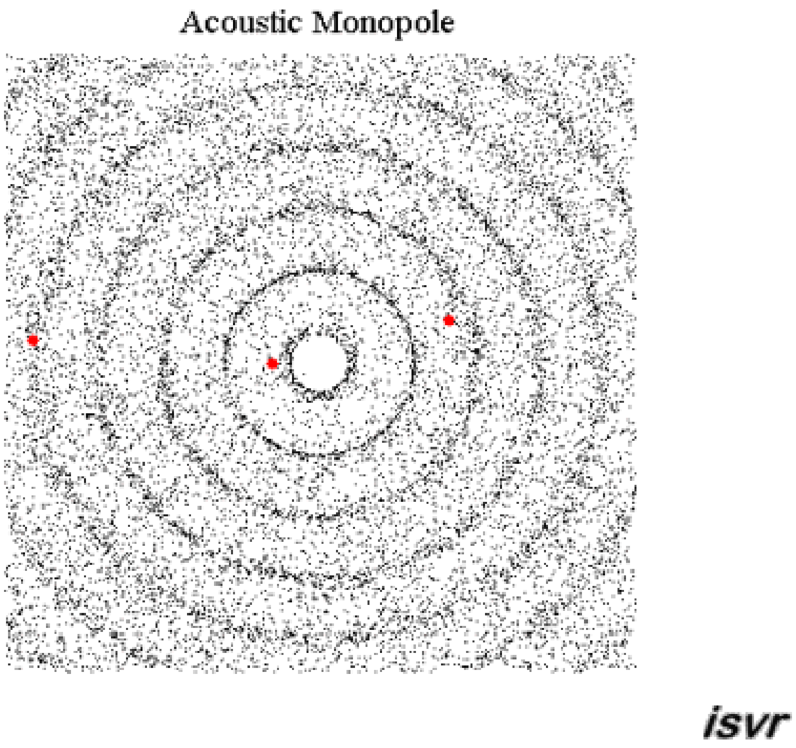






Introduction

Field radiated by a point source / acoustic monopole [1]



[1] http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-basics-pointsources.htm

Heterogeneous media – field equations

Hooke's law: $\nabla \cdot \underline{v}(\underline{r},t) + \kappa(\underline{r})\partial_t p(\underline{r},t) = q(\underline{r},t) \quad \Rightarrow \quad \begin{aligned} &\nabla \cdot \underline{v}(\underline{r},t) + \kappa_0 \partial_t p(\underline{r},t) \\ &\quad = q(\underline{r},t) + \{\kappa_0 - \kappa(\underline{r})\}\partial_t p(\underline{r},t) \end{aligned}$

Newton's law: $\nabla p(\underline{r},t) + \rho(\underline{r})\partial_t \underline{v}(\underline{r},t) = \underline{f}(\underline{r},t) \quad \Rightarrow \quad \begin{aligned} &\nabla p(\underline{r},t) + \rho_0 \partial_t \underline{v}(\underline{r},t) \\ &\quad = \underline{f}(\underline{r},t) + \{\rho_0 - \rho(\underline{r})\}\partial_t \underline{v}(\underline{r},t) \end{aligned}$

Combining the above field equations yields the wave equation

$$\nabla^2 p(\underline{r},t) - \frac{1}{c_0^2}\partial_t^2 p(\underline{r},t) = -\underbrace{\{\rho_0 \partial_t q(\underline{r},t) - \nabla \cdot \underline{f}(\underline{r},t)\}}_{S_{pr}(\underline{r},t)}$$

$$-\underbrace{\{\rho_0(\kappa_0 - \kappa(\underline{r}))\partial_t^2 p(\underline{r},t) - \cancel{\nabla[\{\rho_0 - \rho(\underline{r})\}\partial_t \underline{v}(\underline{r},t)]}\}}_{S_{cs}(\underline{r},t)}$$

$$\boxed{\frac{1}{c^2(\underline{r})} = \rho_0 \kappa(\underline{r})}$$

TUDelft

Multi-parameter Inversion

$$\hat{p}^{sct}(\vec{x}) = \frac{\omega^2}{c_0^2} \int_{\vec{x}' \in \mathbb{D}} \hat{G}(\vec{x} - \vec{x}')\chi^{\kappa}(\vec{x}')\hat{p}^{tot}(\vec{x}')dV(\vec{x}') + i\omega\rho_0 \nabla \cdot \int_{\vec{x}' \in \mathbb{D}} \hat{G}(\vec{x} - \vec{x}')\chi^{\rho}(\vec{x}')\vec{v}^{tot}(\vec{x}')dV(\vec{x}')$$

$$\vec{v}^{sct}(\vec{x}) = i\omega\kappa_0 \nabla \int_{\vec{x}' \in \mathbb{D}} \hat{G}(\vec{x} - \vec{x}')\chi^{\kappa}(\vec{x}')\hat{p}^{tot}(\vec{x}')dV(\vec{x}') - \nabla\nabla \cdot \int_{\vec{x}' \in \mathbb{D}} \hat{G}(\vec{x} - \vec{x}')\chi^{\rho}(\vec{x}')\vec{v}^{tot}(\vec{x}')dV(\vec{x}') + \chi^{\rho}(\vec{x})\vec{v}^{tot}(\vec{x})$$

With the aid of redatuming we can now compute $\vec{v}^{sct}(\vec{x})$ at the location of the receivers

U. Taskin et al, "Redatuming of 2-D Breast Ultrasound," IEEE Trans Ultrason Ferroelectr Freq Control 67(1)
U. Taskin et al, "Multi-parameter inversion with the aid of particle velocity field reconstruction," JASA 47(6)

TU Delft

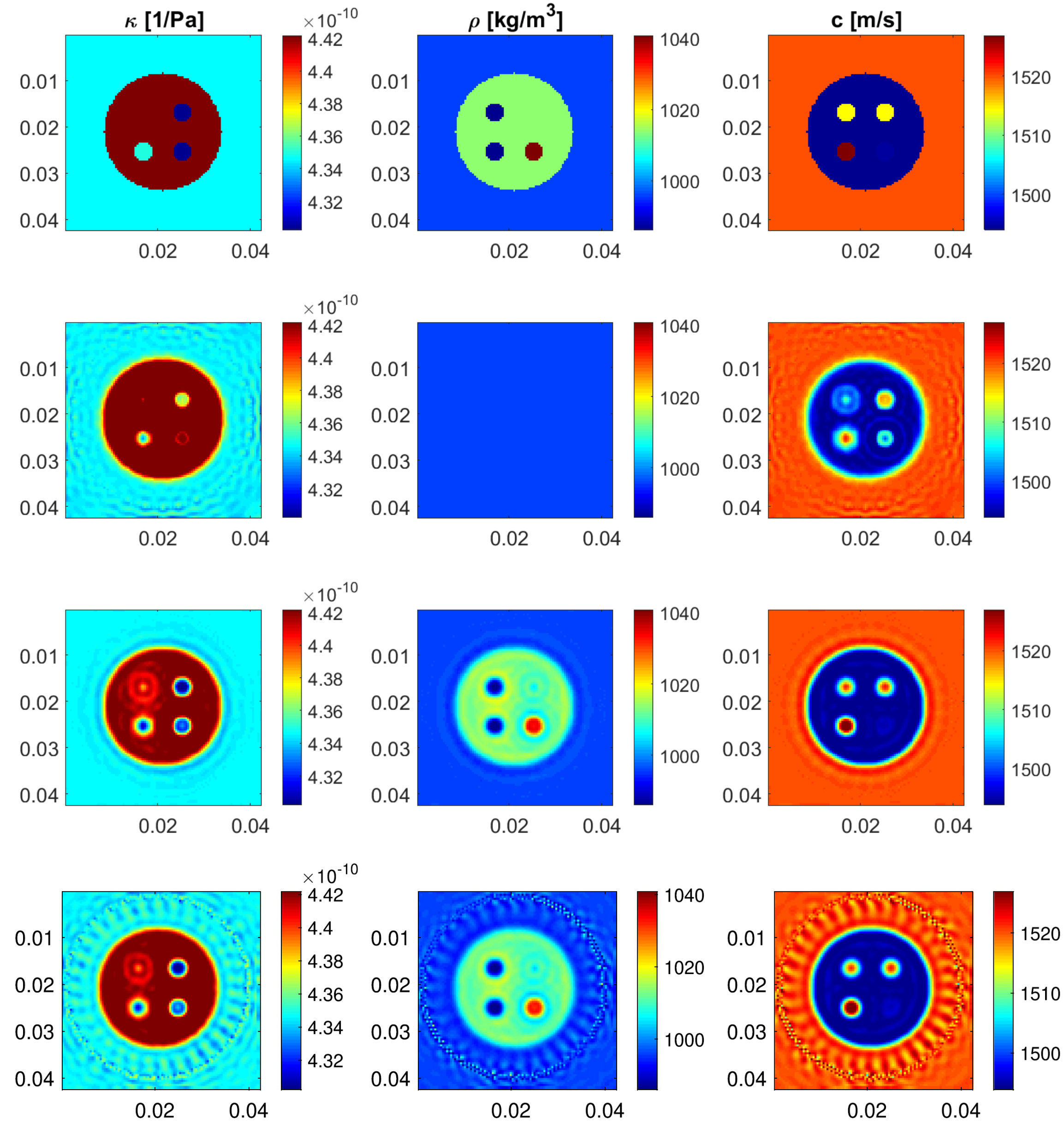
Multi-parameter Inversion

True parameters

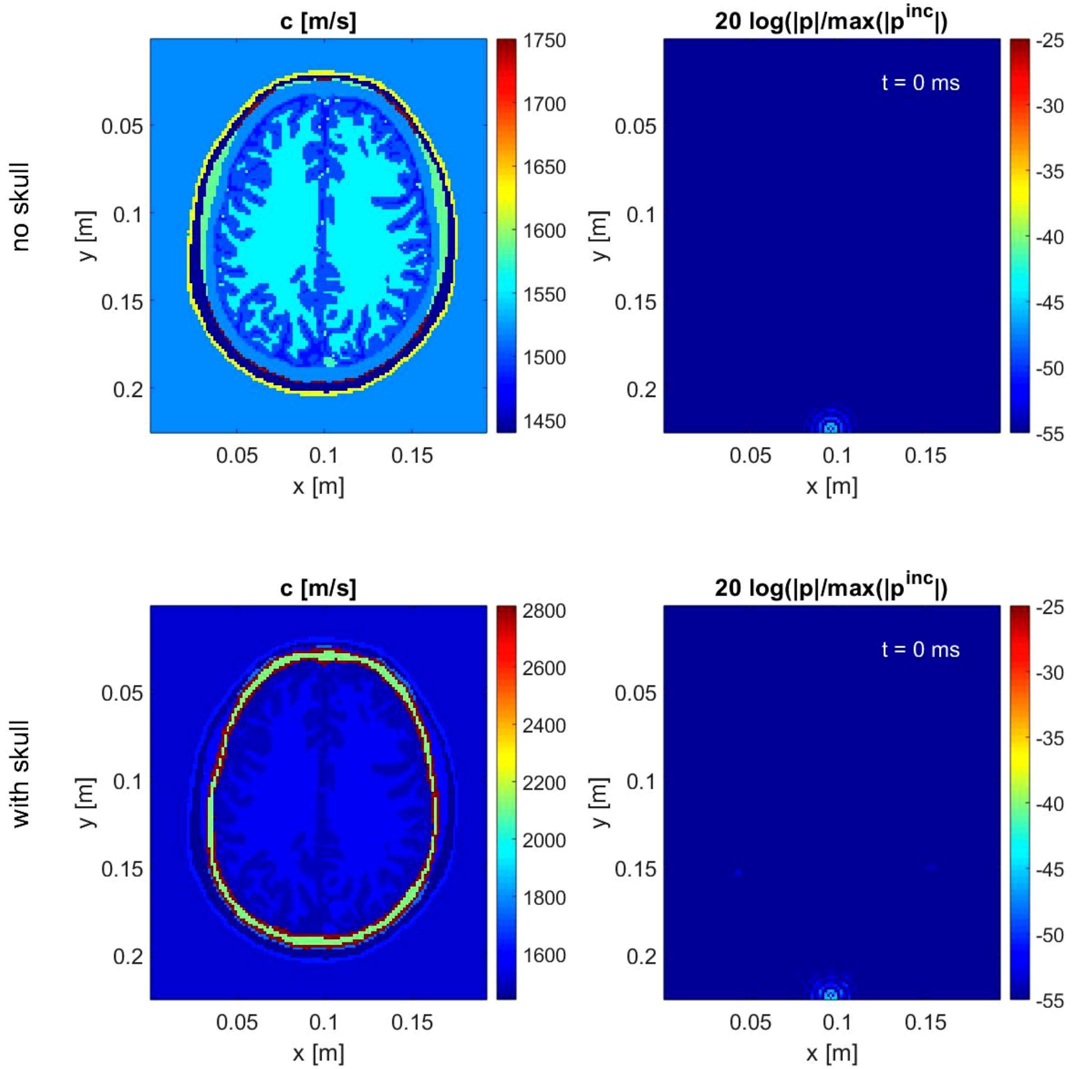
Single parameter

Multi parameter without redatuming

and with redatuming



Transcranial Ultrasound



TU Delft

Multi-parameter / joint inversion

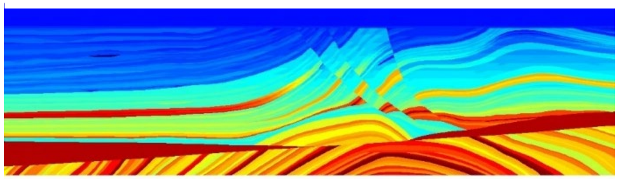
Can we

- *use one reconstruction to enhance the other in an iterative manner?*

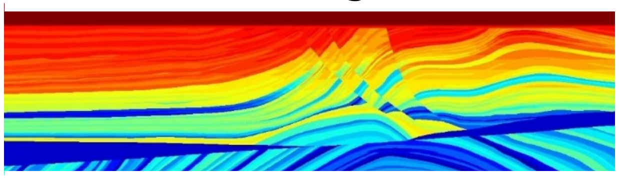
Multi-physics / multi-parameter / joint inversion:

- simultaneous invert for multiple medium parameters,
- regularisation based on structural similarity.

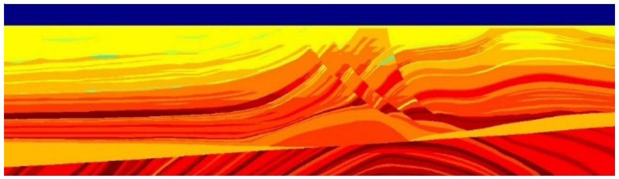
acoustic



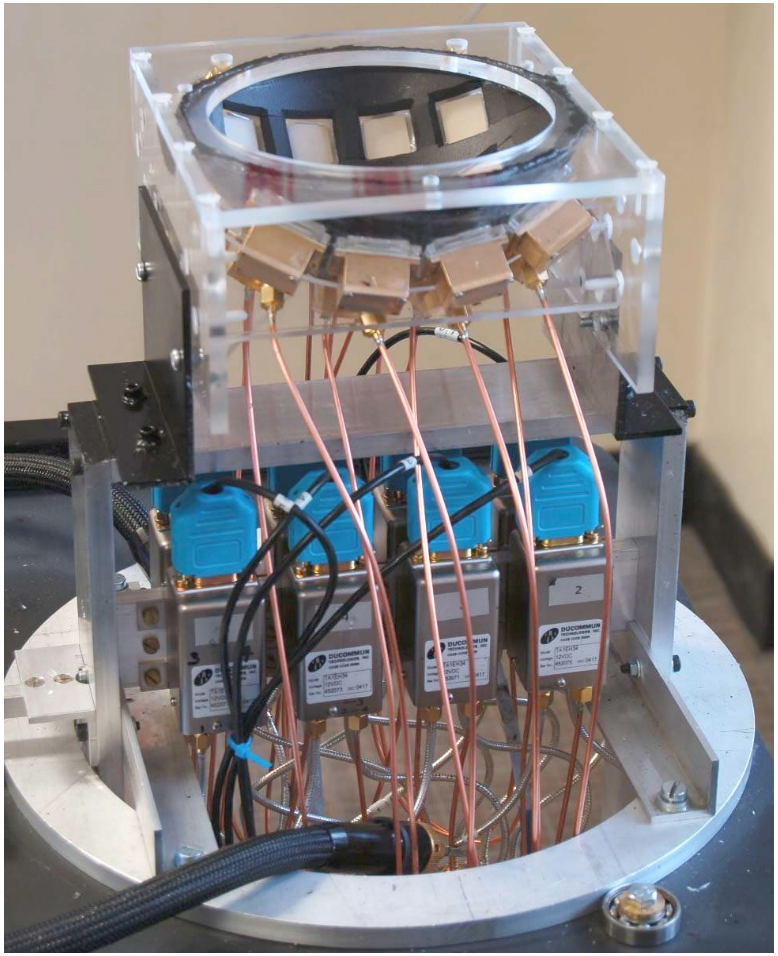
electromagnetic



gravity



Multi-parameter / joint inversion



Klemm et al., "Experimental and clinical results of breast cancer detection using UWB microwave radar,"
2008 IEEE Antennas and Propagation Society International Symposium, 2008.

Multi-parameter / joint inversion

Acoustic

$$\nabla^2 p - \frac{1}{c_{0;AC}^2}\partial_t^2 p = -S_{pr} - \underbrace{\chi_{AC}\partial_t^2 p}_{S_{cs}(\underline{r},t)} \quad \text{with } \chi_{AC} = \frac{1}{c_{0;AC}^2} - \frac{1}{c_{AC}^2(\underline{r})}$$

⇓ Fourier Transform

$$\nabla^2 \hat{p} + \frac{\omega^2}{c_{0;AC}^2}\hat{p} = -\hat{S}_{pr} + \chi_{AC}\omega^2\hat{p}$$

⇓ Integral Equation

$$\hat{p} = \hat{p}^{inc} - \hat{G} *_{\underline{r}} \left[\chi_{AC}\omega^2\hat{p}\right]$$

⇓ Forward / Inverse

Forward problem $\Leftarrow \hat{p}^{inc} = \hat{p} + \hat{G} *_{\underline{r}} \left[\chi_{AC}\omega^2\hat{p}\right] \Leftrightarrow \boxed{p^{inc} = Ap}$

Inverse problem $\Leftarrow \hat{p}^{sct} = -\hat{G} *_{\underline{r}} \left[\chi_{AC}\omega^2\hat{p}^{inc}\right] \Leftrightarrow \boxed{p^{sct} = Ax}$

Electromagnetic TM

$$\nabla^2 \underline{E} - \frac{1}{c_{0;EM}^2}\partial_t^2 \underline{E} = -S_{pr} - \underbrace{\chi_{EM}\partial_t^2 \underline{E}}_{S_{cs}(\underline{r},t)} \quad \text{with } \chi_{EM} = \frac{1}{c_{0;EM}^2} - \frac{1}{c_{EM}^2(\underline{r})}$$

⇓ Fourier Transform

$$\nabla^2 \underline{\hat{E}} + \frac{\omega^2}{c_{0;EM}^2}\underline{\hat{E}} = \hat{S}_{pr} - \chi_{EM}\omega^2\underline{\hat{E}} + \nabla\left[\nabla\cdot\underline{\hat{E}}\right]$$

⇓ Integral Equation

$$\underline{\hat{E}} = \underline{\hat{E}}^{inc} + \hat{G} *_{\underline{r}} \left(\chi_{EM}\omega^2\underline{\hat{E}} - \nabla\left[\nabla\cdot\underline{\hat{E}}\right]\right)$$

⇓ Forward / Inverse

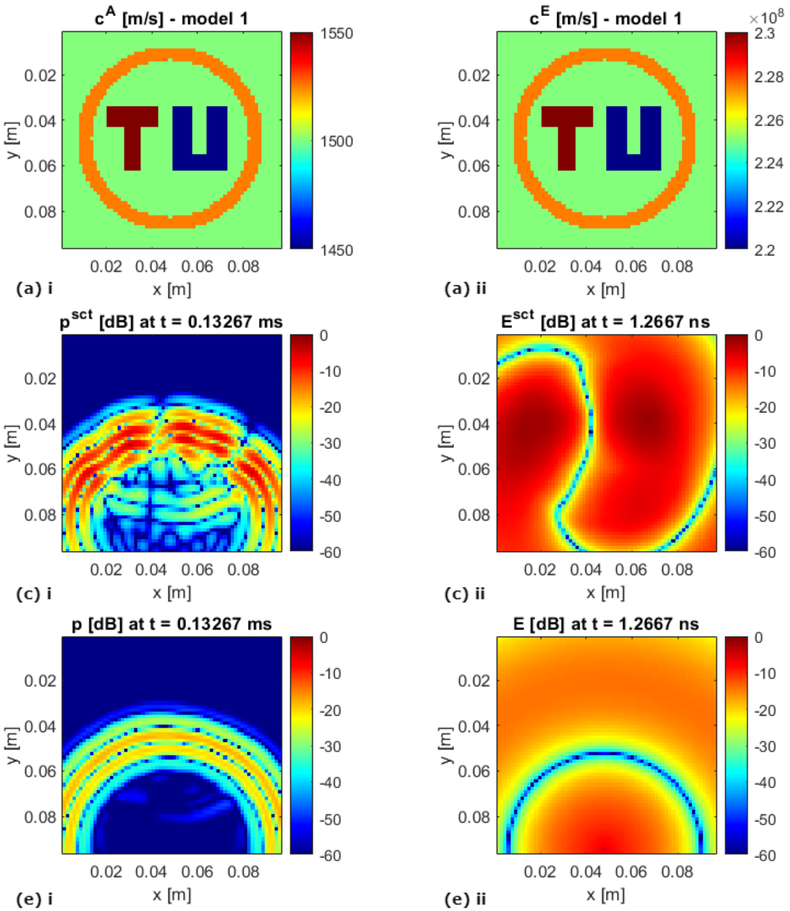
Forward problem $\Leftarrow \underline{\hat{E}}^{inc} = \underline{\hat{E}} + \hat{G} *_{\underline{r}} \left(\chi_{EM}\omega^2\underline{\hat{E}} - \nabla\left[\nabla\cdot\underline{\hat{E}}\right]\right) \Leftrightarrow \boxed{E^{inc} = BE}$

Inverse problem $\Leftarrow \underline{\hat{E}}^{sct}(\underline{r}) = -\hat{G} *_{\underline{r}} \left(\chi_{EM}\omega^2\underline{\hat{E}}^{inc}\right) \Leftrightarrow \boxed{E^{sct} = Bx}$

TU Delft

Multi-parameter / joint inversion

$\lambda_{AC} = 15 \text{ mm} \ll \lambda_{EM} = 225 \text{ mm}$

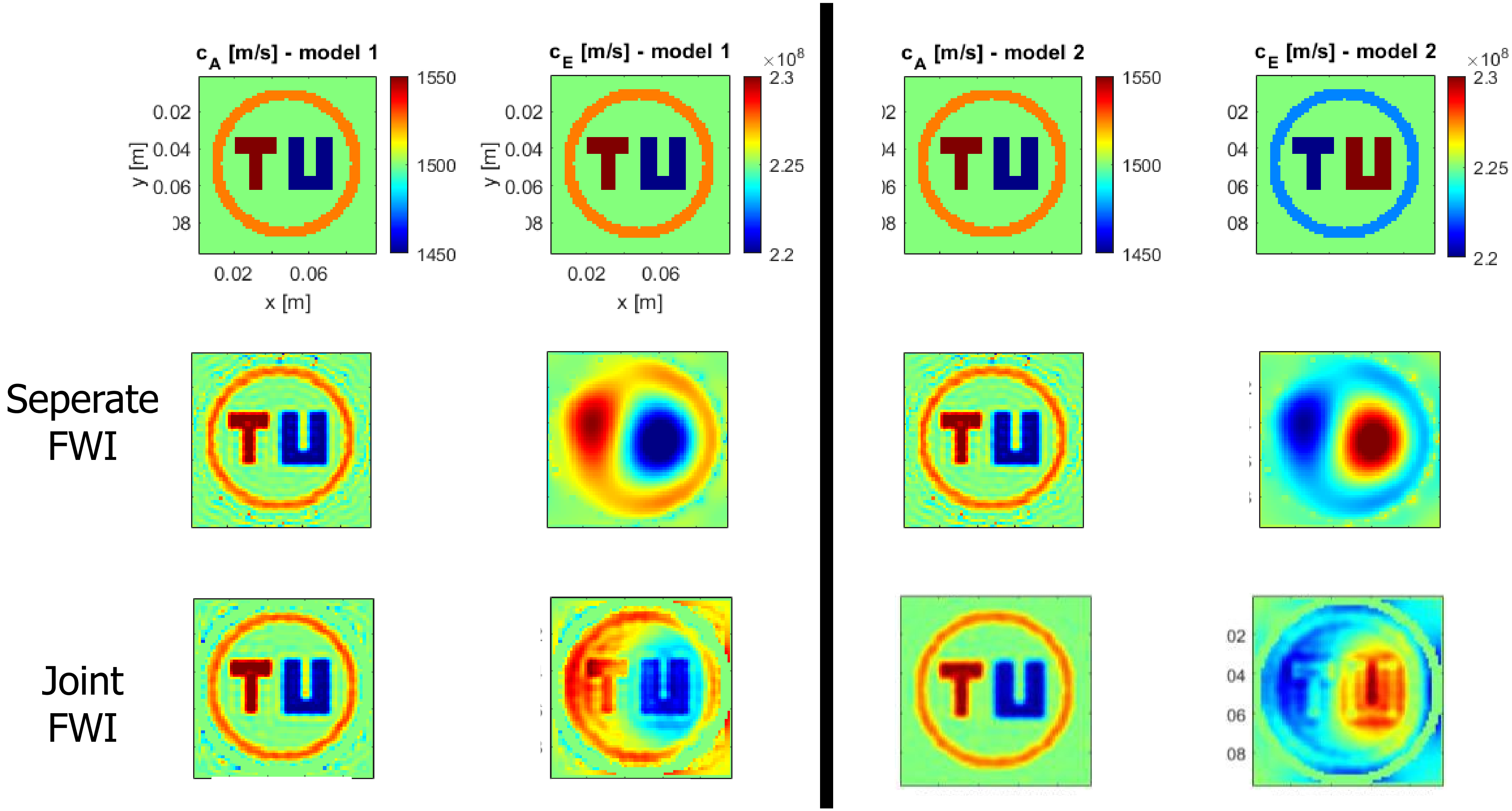


Joint inversion

$$\begin{pmatrix} r_{n;AC} \\ r_{n;EM} \end{pmatrix} = \begin{pmatrix} p^{sct}_{meas} \\ E^{sct}_{meas} \end{pmatrix} - \begin{pmatrix} G^{AC}\left[\chi_{n;AC}\right] \\ G^{EM}\left[\chi_{n;EM}\right] \end{pmatrix}$$

$$\begin{pmatrix} \chi_{n;AC} \\ \chi_{n;EM} \end{pmatrix} = \begin{pmatrix} \chi_{n-1;AC} \\ \chi_{n-1;EM} \end{pmatrix} + \begin{pmatrix} \alpha_{n;AC} d_{n;AC} \\ \alpha_{n;EM} d_{n;EM} \end{pmatrix}$$

$$\Leftrightarrow \quad Err_n = \frac{\left\| \begin{pmatrix} r_{n;AC} \\ r_{n;EM} \end{pmatrix} \right\|^2}{\left\| \begin{pmatrix} p^{sct}_{meas} \\ E^{sct}_{meas} \end{pmatrix} \right\|^2} + \beta \left\| \nabla \chi_{n;AC} - \nabla \chi_{n;EM} \right\|^2$$



Discussion and conclusion

SAFT
- Echogenicity
- High frequency

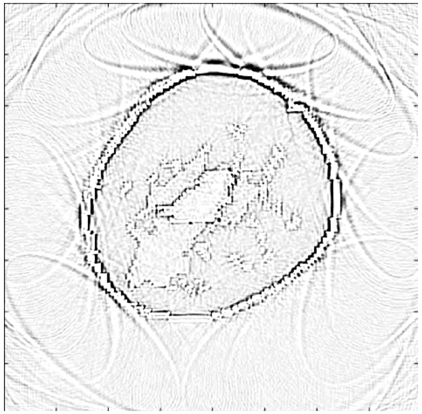
Geometric / Ray based methods
- Speed of sound and attenuation
- High frequency
- Many src/rec combinations

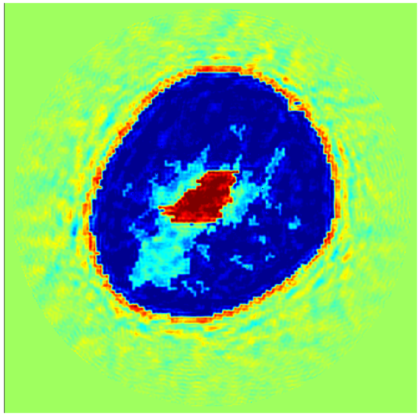
Born Inversion / Parabolic methods
- 2D - Neglects multiple scattering and phase shifts
- "Speed of sound & attenuation"
- Convergence

Full-Wave Form Inversion
- Taking advantage of multiple scattering and phase shifts
- Speed of sound & attenuation
- 3D - Computational heavy -> Low frequencies only

Machine learning

Multi-parameter / Joint Inversion





Thank you very much attention